AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by J. H. CURTISS

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Please send in abstracts of papers to be presented in person well in advance of the deadline.

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
553	January 20-22, 1959 (65th Annual Meeting)	Philadelphia, Pennsylvania	Dec. 5
554	February 28, 1959	New York, New York	Jan. 15
555	April 17-18, 1959	Chicago, Illinois	Mar. 4
556	April 17-18, 1959	Monterey, California	Mar. 4
557	April 23-25, 1959	New York, New York	Mar. 4
558	June 20, 1959	Eugene, Oregon	May 7
559	September 2-5, 1959 (64th Summer Meeting)	Salt Lake City, Utah	July 20
560	October 31, 1959	Cambridge, Massachusetts	Sept. 17
	November, 1959	Detroit, Michigan	
	January 27-29, 1960 (66th Annual Meeting)	Chicago, Illinois	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, Ann Arbor, Michigan, or 190 Hope Street, Providence 6, R. I.

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FIVE HUNDRED FIFTIETH MEETING

Pomona College
Claremont, California
November 21-22, 1958

PROGRAM

The five hundred fiftieth meeting of the American Mathematical Society will be held on Friday and Saturday, November 21-22, 1958, at Pomona College, Claremont, California. All sessions will meet in the Physics-Mathematics Building.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, a Symposium on Asymptotic Expansions will be held on Friday in the Auditorium of the Physics-Mathematics Building, with sessions at 10:00 A.M. and 2:00 P.M.

By invitation of the same Committee, Professor Leon Henkin of the University of California, Berkeley, will address the Society on Saturday at 1:30 P.M. His topic is "Some recent work on real closed fields".

Sessions for contributed papers will be held on Saturday at 10:00 A.M. in room 134 and the auditorium, and at 3:00 P.M. in the auditoium of the Physics-Mathematics Building. If necessary, there will be a session for late papers at 3:00 P.M. in room 134. For information on late papers inquire at the registration desk which will be located in the patio at the main entrance of the Physics-Mathematics Building. Registration will begin at 9:00 A.M. on Friday and Saturday.

Tea will be served on Friday in the Harvey Mudd Residence Hall on the Harvey Mudd College Campus, and on Saturday, following the sessions for contributed papers, in Walker Lounge of the Men's Dormitory on the Pomona College Campus. A recital will be presented by the Claremont Trio at 8:00 P.M. on Friday in the Bridges Hall of Music.

Midday dinner on Saturday will be available at the Gibson Dining Hall on Campus. There are numerous restaurants in and near Claremont.

Claremont can be reached by bus from Sixth and Main Streets in Los Angeles. Private cars coming from Los Angeles should take the San Bernardino Freeway (U.S. 99) beyond the city of Pomona to Alexander Avenue in Claremont and then follow signs to Pomona College. The Santa Fe Railroad has a stop in North Pomona, and Southern Pacific in Pomona. The Pomona College campus is a short taxi ride from either of these stations.

PROGRAM OF THE SYMPOSIUM ON ASYMPTOTIC EXPANSIONS

The program committee consisted of Professors Arthur Erdélyi, chairman, J. G. van der Corput, R. E. Langer, and T. E. Hull.

Both sessions of the Symposium will meet in the auditorium of the Physics-Mathematics Building.

FRIDAY, 10:00 A.M.

Invited Addresses

On the Theory of Neutrices
Professor J. G. van der Corput, University of California, Berkeley

Asymptotic Solutions of Ordinary Linear Differential Equations with Respect to a Parameter
Professor R. E. Langer, University of Wisconsin

FRIDAY, 2:00 P.M.

Invited papers

On Integral Equations with Asymptotic Liouville Neumann Series
Professor T. E. Hull, University of British Columbia and California Institute of Technology

Saddle-point Integrals with Neighboring Saddle Points
Professor B. Friedman, University of California, Berkeley

Recent Advances and Open Questions on the Asymptotic Expansions of Orthogonal Polynomials
Professor G. Szegö, Stanford University

The Asymptotic Behavior of the Laurent Coefficients
Professor Max Wyman, University of Alberta

Simplification of Systems of Linear Differential Equations Involving a Turning Point
Professor H. L. Turrittin, University of Minnesota

Linear Differential Equations of the Second Order with a Large Parameter
Mr. F. W. J. Oliver, National Physical Laboratory (England) and National Bureau of Standards

PROGRAM OF THE SESSIONS
(Time limit for each contributed paper, 10 minutes)

SATURDAY, 10:00 A.M.

Session on Logic and Foundations, Auditorium, Physics-Mathematics Building

(1) Some model-theoretical results concerning weak second-order logic
Professor Alfred Tarski, University of California, Berkeley (550-6)

(2) Set-theoretical properties of reduced products
Mr. T. E. Frayne, University of California, Berkeley, Professor Anne C. Morel, University of California, Davis, and Dr. D. S. Scott, University of Chicago (550-8)

(3) Reduced products and the compactness theorem
Professor Anne C. Morel, University of California, Davis, Dr. D. S. Scott, University of Chicago, and Professor Alfred Tarski, University of California, Berkeley (550-7)

(4) Model-theoretical properties of reduced products
Mr. T. E. Frayne, University of California, Berkeley, and Dr. D. S. Scott, University of Chicago (550-9)

(5) Arithmetically definable models of formalized arithmetic
Professor Solomon Feferman, Stanford University (550-21)

(6) Remarks on isolic arithmetic. II
Professor Anil Nerode, University of California, Berkeley (550-16)

(7) Theories having at least continuum many nonisomorphic models in each infinite power
Mr. Andrzej Ehrenfeucht, University of California, Berkeley (550-23)
(Introduced by Professor R. L. Vaught)

(8) Filtration systems. II
Professor S. B. Kochen, University of Montreal (550-2)

Session on Analysis, Applied Mathematics and Topology, Room 134, Physics-Mathematics Building

(9) The structure of electrical networks
Dr. W. T. Kyner and Dr. Milton Lees, Shell Development Company, Emeryville, California (550-17)

(10) Some nonoscillation theorems on the real axis for complex differential equations
Professor D. V. V. Wend, University of Utah (550-3)

(11) Absolutely continuous solutions of a certain nonlinear hyperbolic partial differential equation
Professor D. H. Tucker, University of Utah (550-15)

(12) Infinitely differentiable solutions of ordinary differential equations
Dr. H. P. Flatt, Atomics International, Inc., Canoga Park, California (550-24)

(13) Some conditions under which a homogeneous continuum is a simple closed curve
Professor C. E. Burgess, University of Utah (550-11)

(14) Minimal surfaces with bounded normal direction
Professor Robert Osserman, Stanford University (550-25)

(15) Some boundary properties of maximal algebras
Professor H. S. Bear, University of Washington (550-13)

(16) Projections onto the subspace of compact operators
Mr. E. O. Thorp, University of California, Los Angeles (550-4)

(17) Banach spaces with a specified number of separable conjugate spaces
Professor R. C. James, Harvey Mudd College (550-22)

SATURDAY, 1:30 P.M.

Invited Address, Auditorium, Physics-Mathematics Building
Some recent work on real closed fields
Professor Leon Henkin, University of California, Berkeley

SATURDAY, 3:00 P.M.

Session on Algebra, Logic and Foundations, Auditorium, Physics-Mathematics Building

(18) The nucleus of a torsion-free abelian group
Professor R. A. Beaumont, University of Washington, and Professor R. J. Wisner, Haverford College (550-1

(19) Positive bases for linear spaces
Dr. R. L. McKinney, University of California, Riverside (550-18)

(20) Variations on a theme of Chevalley
Professor Robert Steinberg, University of California, Los Angeles (550-5)

(21) Structure of cleft rings. I
Professor J. H. Walter, University of Washington (550-19)

(22) Universal relational systems for elementary classes and types
Professor R. L. Vaught, University of California, Berkeley (550-1)

(23) Split semantic models
Mrs. Carol R. Karp, University of Maryland (550-20)

(24) The theory of models with generalized atomic formulas. I
Mr. H. J. Keisler, California Institute of Technology (550-14)
(Introduced by Dr. Olga Taussky)

SUPPLEMENTARY PROGRAM
(To be presented by title)

(25) On the solution of an nth order implicit differential equation
Professor Smbat Abian and Professor A. B. Brown, Queens College

(26) Combinatory methods in the general theory of queues
Dr. V. E. Beněs, Bell Telephone Laboratories, Murray Hill, New Jersey

(27) Reduced products
Mr. T. E. Frayne, University of California, Berkeley, Dr. D. S. Scott, University of Chicago, and Professor Alfred Tarski, University of California, Berkeley

(28) Homogeneous universal relational systems
Professor Bjarni Jónsson, University of Minnesota

(29) The theory of models with generalized atomic formulas. II
Mr. H. J. Keisler, California Institute of Technology
(Introduced by Dr. Olga Taussky)

(30) A numerical method for solving the torsion problem of a bar
Professor E. O. A. Kreyszig, Ohio State University

(31) Simple error bounds for eigenvalues and eigenvectors of hermitian matrices
Dr. Joachim Nitsche, Universitaet Freiburg i. Br.
(Introduced by Professor J. C. C. Nitsche)

(32) On the Löwenheim-Skolem theorem in weak second-order logic
Dr. D. S. Scott, University of Chicago

(33) Extension principles for algebraically closed fields
Dr. D. S. Scott, University of Chicago, and Professor Alfred Tarski, University of California, Berkeley

(34) On the group of invariant automorphisms of a Lie algebra
Professor G. B. Seligman, Yale University and University of Muenster

(35) Universal spaces for some metrisable uniformities
Dr. A. H. Stone, University of Manchester

(36) Homogeneous universal models of complete theories
Professor R. L. Vaught, University of California, Berkeley

(37) Prime models and saturated models
Professor R. L. Vaught, University of California, Berkeley

R. S. Pierce
Acting Associate Secretary

Seattle, Washington
October 14, 1958

FIVE HUNDRED FIFTY-FIRST MEETING

Duke University
Durham, North Carolina
November 28-29, 1958

PROGRAM

The five hundred fifty-first meeting of the American Mathematical Society will be held at Duke University, Durham, North Carolina, on November 28-29, 1958, in the Physics Building on West Campus.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor E. E. Floyd of the University of Virginia will address the Society on "Compact groups of transformations" at 2:00 P.M. Friday in Room 114, Physics.

Sessions for contributed papers will be held at 3:30 P.M. to 5:00 P.M. on Friday and 10:00 A.M. to noon on Saturday.

Registration headquarters will be in the Physics Building, and will be open from 10:00 A.M. Friday through noon Saturday. Tea will be served at 4:00 P.M. Friday. There will be a banquet Friday at 6:30 P.M., the charge being $2.00 per person. A cafeteria in the Union Building on the campus will be available for breakfast for the exclusive use of members of the Society and their guests on Saturday morning, from 8:00 to 9:00 A.M. Coffee will also be served Saturday morning in the Physics Building. Banquet reservations should be made by card or letter to Professor R. M. McLeod, Department of Mathematics, Duke University, Durham, North Carolina, and should reach him by November 14. Reservations for rooms should be made directly with the chosen hotel or motel. (Downtown Durham is approximately 3 miles from the West Campus of Duke University.) Bus service is available between downtown Durham and West Campus. Ample parking space is available near the Physics Building. Minimum rates are listed below for conveniently located hotels and motels:

	Single	Double
Washington Duke Hotel Corcoran and Chapel Hill Sts., Durham	$4.50	$8.25
Biltmore Hotel (colored) 332 East Pettigrew, Durham	3.00	4.00
Eden Rock Motel, 3 miles South of Durham on Highways 15A and 501(3 1/2 miles from the Physics Building)	---	10.00

	Single	Double
Homestead Motel, 2 miles South of Durham on Highways 15A and 501 (3 miles from Physics Building)	$7.00	$8.00
El Rancho Motel, Elf Street, Durham near Veterans' Hospital (3/4 miles from Physics Building)	7.00	9.00

Mail and telegrams for those attending the meeting may be sent in care of the Mathematics Department, Physics Building, Duke University, Durham, North Carolina.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

FRIDAY, 2:00 P.M.

General Session, Room 114, Physics Building

Compact groups of transformations (One hour)
Professor E. E. Floyd, University of Virginia

FRIDAY, 3:30 P.M.

Session on Topology, Room 114, Physics Building

(1) On a problem concerning local connectedness in the sense of singular homology
Dr. Sibe Mardešić, Institute for Advanced Study (551-8)

(2) Imbedding decompositions of 3-space in 4-space
Professor R. H. Bing, University of Wisconsin, and Professor M. L. Curtis, University of Georgia (551-9)

(3) Euclidean domains with uniformly abelian local fundamental groups. III
Professor O. G. Harrold, Jr., University of Tennessee (551-12)

(4) e-mapping of a disc onto a torus
Professor M. K. Fort, Jr., University of Georgia (551-10)

(5) Homogeneity of inverse limit spaces
Mr. Jack Segal, University of Georgia (551-15)

(6) Arcs in partially ordered spaces
Professor R. J. Koch, Louisiana State University (551-16)

(7) Topological completeness of a space of homeomorphisms
Professor M. K. Fort, Jr., and Mr. W. G. Horstman, University of Georgia (551-18)

(8) Certain one dimensional semigroups
Professor R. P. Hunter, University of Georgia (551-24)
(Introduced by Professor G. B. Huff)

(9) Continua with sets of indecomposability
Professor P. M. Swingle, University of Miami (551-20)

Session on Applied Mathematics, Room 113, Physics Building

(10) A general algorithmic method for routing and network problems
Mr. G. J. Minty, University of Michigan (551-1)

(11) Criteria for orbital stability
Dr. J. K. Hale, RIAS, Inc., Baltimore, Maryland (551-11)

(12) Stability of two-parameter systems
Professor Andrew Sobczyk, University of Florida (551-14)

(13) On singular solutions of certain non-linear differential equations from the theory of interior ballistics. Preliminary report
Professor R. A. Struble, North Carolina State College (551-26)

SATURDAY, 10:00 A.M.

Session on Analysis, Room 114, Physics Building

(14) A topological proof of the continuity of the derivative of a function of a complex variable
Professor R. L. Plunkett, Florida State University (551-2)

(15) Some convergence theorems for continued fractions
Mr. Robert Heller, Jr., University of Texas (551-3)

(16) On the almost periodicity of the integral of an almost periodic function
Dr. G. H. Meisters, Duke University (551-4)

(17) Interior-like elements of cones. Preliminary report
Professor C. C. Braunschweiger, University of Delaware (551-6)

(18) A problem in linear differential equations
Professor Tomlinson Fort, University of South Carolina (551-7)

(19) On the existence of the fundamental solution of the parabolic equation
Mr. S. M. Robinson, Duke University (551-13)

(20) On the compatibility of systems of linear differential equations

Professor E. J. Pellicciaro, University of Delaware (551-22)

(21) On total stability

Professor Lamberto Cesari, Purdue University (551-23)

(22) Closed form solutions of $z'' + (p + u)z' + (q + v)z = 0$ in terms of solutions of $y'' + py' + qy = 0$ and of known functions

Professor J. W. Cell, North Carolina State College (551-27)

Session on Algebra and Theory of Numbers, Room 113, Physics Building

(23) Some consequences of Wilson's theorem

Dr. D. O. Ellis, Litton Industries, Beverly Hills, California (551-5)

(24) Zonal functions of the representation of the linear group in the space of polynomials of a symmetric matrix. Preliminary report

Dr. A. T. James, Yale University (551-17)

(25) Compact principal domains

Professor S. L. Warner, Duke University (551-19)

(26) Integral domains defined on torsion-free groups of rank two

Professor R. A. Beaumont, University of Washington, and Professor R. J. Wisner, Haverford College (551-2

(27) Some finite summation formulas of arithmetic character

Professor Leonard Carlitz, Duke University

(28) Note on the number of pairs of non-intersecting permutations

Professor Jack Levine, North Carolina State College (551-25)

SUPPLEMENTARY PROGRAM

(To be presented by title)

(29) A class of arithmetical functions of several variables. I. Applications to congruences

Professor Eckford Cohen, University of Tennessee

(30) Arithmetical functions associated with arbitrary sets of integers

Professor Eckford Cohen, University of Tennessee

(31) Asymptotic averages in a class of arithmetical functions of two variables
Professor Eckford Cohen, University of Tennessee

(32) Singular homology of one-dimensional spaces
Professor M. L. Curtis and Professor M. K. Fort, Jr., University of Georgia

(33) One-to-one mappings onto the Cantor set
Professor M. K. Fort, Jr., University of Georgia

(34) Images of plane continua
Professor M. K. Fort, Jr., University of Georgia

(35) Some determinantal equations related to representations by forms over a finite field
Dr. J. H. Hodges, Cornell Aeronautical Laboratory, Inc.

(36) Ordered semigroups in partially ordered semigroups
Professor R. J. Koch, Louisiana State University

(37) Concerning the weak cutpoint ordering
Professor R. J. Koch and Professor I. S. Krule, Louisiana State University

(38) Dimension and inverse limits of compact spaces
Dr. Sibe Mardešic, Institute for Advanced Study

G. B. Huff
Associate Secretary

Athens, Georgia
October 16, 1958

FIVE HUNDRED FIFTY-SECOND MEETING

Northwestern University
Evanston, Illinois
November 28-29, 1958

PROGRAM

The five hundred fifty-second meeting of the American Mathematical Society will be held at Northwestern University, Evanston, Illinois, on Friday and Saturday, November 28-29, 1958. All sessions will be held in the Technological Institute.

The Committee to Select Hour Speakers for Western Sectional Meetings has invited Professor Roger Lyndon of the University of Michigan to address the Society. Professor Lyndon will speak on "Properties preserved under algebraic operations", at 2:00 P.M. in Lecture Room 2 of the Northwestern Technological Institute Building.

Sessions for the presentation of contributed papers will be held at 3:15 P.M. on Friday, and at 10:30 A.M. on Saturday. If necessary, there will be special sessions for the presentation of contributed papers which failed to meet the deadline. If such sessions are held, they will be publicized by a special program available at the registration desk.

The registration desk will be located in the front lobby of the Northwestern Technological Institute Building located on Sheridan Road at Noyes Street. Those who attend the meetings are requested to register at any time from 9:30 A.M. to 5:00 P.M. on Friday, and from 9:30 A.M. to noon Saturday.

Members are requested to park in one of the two University parking lots directly across Sheridan Road from the Technological Institute.

The dining room of Sargent Hall, directly north of the Technological Institute, will be open for all meals.

The Orrington Hotel, 1710 Orrington Avenue, Evanston, has agreed to hold a block of rooms for Society members until November 13, 1958. Members should write directly to the Orrington. Single rooms range from $7.00 to $12.00 and double rooms from $10.00 to $14.00. The North Shore Hotel in Evanston has comparable prices. There is also a Y.M.C.A. available.

Mail and telegrams for those attending the meeting should be addressed in care of the Mathematics Department, Lunt 216, Northwestern University, Evanston, Illinois.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

All Sessions in Northwestern Technological Institute Building

FRIDAY, 2:00 P.M.

General Session, Lecture Room 2

Properties preserved under algebraic operations (One hour)
Professor Roger Lyndon, University of Michigan

FRIDAY, 3:15 P.M.

General Session, Lecture Room 2

(1) Each homogeneous plane continuum that contains an arc separates the plane
Professor R. H. Bing, University of Wisconsin (552-13)

(2) Inversive geometry, complex numbers and proportionality
Professor G. Y. Rainich, University of Notre Dame (552-18)

(3) Local cyclic connectedness. I. Cyclic subelements of cyclic elements
Professor L. F. McAuley, University of Wisconsin (552-17)

(4) Compactifications of dimension zero
Professor L. J. Heider, Marquette University (552-9)

(5) A resolution of the identity for certain unbounded linear operators
Dr. M. A. Martino, Jr., General Electric Company, Schenectady, New York (552-16)

SATURDAY, 10:30 A.M.

General Session I, Lecture Room 2

(6) On periodicity of states in linear modular sequential circuits
Professor Bernard Friedland and Professor T. E. Stern, Columbia University (552-11)
(Introduced by Professor L. A. Zadeh)

(7) On the eigenvalues and eigenvectors of a class of matrices
Professor S. V. Parter, Indiana University (552-12)

(8) Completeness of quotient algebras of Boolean algebras. III
Professor Philip Dwinger, Purdue University (552-2)

(9) On an extension of a classical theorem on totally ordered groups
Professor N. L. Alling, Purdue University (552-3)

(10) On the norm of a group
Professor E. V. Schenkman, University of Wisconsin (552-14)

(11) Frobenius groups and a related problem
Dr. J. G. Thompson, University of Chicago (552-8)

General Session II, Lecture Room 3

(12) The divergence set of Rademacher series
Mr. W. A. Beyer, Pennsylvania State University (552-10)

(13) The Fourier coefficients of automorphic forms on horocyclic groups. II
Professor Joseph Lehner, Michigan State University (552-7)

(14) On meromorphic functions of finite order
Professor S. M. Shah, University of Wisconsin (552-4)

(15) A note on perturbation theory of ordinary differential operators. Preliminary report
Professor J. B. Butler, Jr., University of Washington (552-1)

(16) On stochastic linear inequalities
Professor J. V. Talacko, Marquette University (552-1

(17) An extension of the renewal theorem
Professor F. L. Spitzer, University of Minnesota (552-6)

(18) Order statistics of partial sums
Professor J. G. Wendel, University of Michigan (552-

SUPPLEMENTARY PROGRAM
(To be presented by title)

(19) On the solution of simultaneous implicit equations
Professor Smbat Abian and Professor A. B. Brown, Queens College

(20) On the openness and inversion of differentiable mappings
Professor R. G. Bartle, University of Illinois

(21) Inversion of primitive functions (mod r) and Nagell's totient function
Professor Eckford Cohen, University of Tennessee

(22) On fiber homotopy equivalence
Professor E. R. Fadell, University of Wisconsin

J. W. T. Youngs
Associate Secretary

Bloomington, Indiana
October 8, 1958

PRELIMINARY ANNOUNCEMENT OF MEETING

SIXTY-FIFTH ANNUAL MEETING

University of Pennsylvania
Philadelphia, Pennsylvania
January 20-22, 1959

The sixty-fifth Annual Meeting of the American Mathematical Society will be held on Tuesday, Wednesday, and Thursday, January 20-22, 1959, at the University of Pennsylvania in Philadelphia, Pennsylvania, in conjunction with meetings of the Mathematical Association of America on Thursday and Friday, January 22 and 23, the Association for Symbolic Logic on Thursday, January 22, and the Delaware Valley Section of the Society for Industrial and Applied Mathematics on Wednesday, January 21.

The thirty-second Josiah Willard Gibbs Lecture will be delivered by Professor J. M. Burgers of the University of Maryland on Tuesday at 8:00 P.M. in the auditorium of the University Museum. The title of the lecture is "On the emergence of patterns of order."

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, Professor G. D. Mostow of Johns Hopkins University will deliver an address on "Compact transformation groups" on Tuesday at 2:00 P.M., and Professor Felix Browder of Yale University will address the Society on "The spectral theory of partial differential operators" on Wednesday at 2:00 P.M. Both addresses will be given in the auditorium of the University Museum.

The award of the Bôcher Prize will be announced on Thursday at 2:00 P.M. in the auditorium of the University Museum. This award will be followed by the Annual Business Meeting.

Sessions for contributed papers will be held in the Physical Sciences Building and the Moore School on Tuesday, Wednesday, and Thursday at 10:00 A.M. and 3:15 P.M.

There will be a banquet on Thursday evening at the Benjamin Franklin Hotel.

The Council of the Society will meet on Wednesday at 5:00 P.M. in the Benjamin Franklin Hotel.

The Employment Register will be maintained in the Bishop White room of Houston Hall throughout the meeting.

REGISTRATION

Registration headquarters will be in the Foyer of Houston Hall (Spruce Street, between 34th and 36th Streets). The desk will be maintained on Tuesday through Thursday from 9:00 A.M. to 5:00 P.M. and on Friday from 9:00 A.M. to 2:00 P.M. In accordance with the So-

ciety's recent decision to charge registration fees at Annual Meetings as well as at Summer Meetings, there will be a registration fee of $1.00 for each member of any participating organization and $ .50 for each accompanying adult.

ROOMS AND MEALS

Accommodations will be available at hotels. The following hotels, listed in order of increasing distance from the University of Pennsylvania (the first two are within walking distance), have agreed to set aside blocks of rooms for those attending the meeting:

Hotel Normandie, 36th and Chestnut Streets, Philadelphia 4. (Rates: singles $6.00 - 7.50; doubles $8.00 - 9.50; twin bedrooms $9.50 - 11.00; rooms for three $12.00; rooms for four $15.00).

The Penn Sherwood, 39th and Chestnut Streets, Philadelphia 1. (Rates: singles $7.00; doubles $10.00; twin bedded rooms $13.00).

Sheraton Hotel, 1725 Pennsylvania Boulevard, Philadelphia 3. (Rates: singles $8.50 - 18.50; doubles $13.00 - 15.00; twin bedded rooms $15.00 - 22.00; rollaway beds $3.00).

The Benjamin Franklin Hotel, 9th and Chestnut Streets, Philadelphia 5. (Rates: singles $6.00; doubles or twin bedrooms $10.00; rooms for three $4.50 per person).

Members should make their own reservations not later than January 5, 1959. In their letters of reservation they should refer to the meeting of the American Mathematical Society, as some of the quoted prices are special rates for members of the participating organizations only.

Meals can be taken at several cafeterias on the University of Pennsylvania campus. A list of restaurants close to the campus and downtown will be distributed at the registration desk.

ENTERTAINMENT AND RECREATION

There will be an official reception by the University of Pennsylvania on Wednesday from 4:00 to 6:00 P.M.

A social get-together, sponsored by the Delaware Valley section of the Society for Industrial and Applied Mathematics, will be held on Wednesday at 9:00 P.M. in the Rotunda of the University Museum. Tickets for $1.00 will be available at the Registration Desk.

The banquet will be held at the Benjamin Franklin Hotel on Thursday evening. Tickets for the banquet ($5.00 each) should be obtained at the time of registration.

Conducted tours of the University campus including the Computer Center and University Museum, and possibly of Philadelphia, will be organized in case there is interest in them. Maps of the cam

pus, maps of Philadelphia, lists of restaurants, addresses for shopping downtown and sites of interest in Philadelphia will be available at the Registration Desk.

Committee on Arrangements:

P. A. Caris
J. H. Curtiss
Robert Ellis
H. M. Gehman
W. H. Gottschalk
R. D. Schafer
G. E. Schweigert
C. T. Yang
Emil Grosswald, Chairman

Further details of the meeting will appear in the next issue of the NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, December 5. It is expected that abstracts of all papers to be presented in person will appear in the same issue of the NOTICES, but only abstracts which meet the specifications stated on the abstract blanks can be published.

R. D. Schafer
Associate Secretary

Princeton, New Jersey
October 8, 1958

ACTIVITIES OF OTHER ASSOCIATIONS

THE FORTY-SECOND ANNUAL MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA will be held at the University of Pennsylvania, Philadelphia, Pennsylvania, on January 22 and 23, 1959. Sessions of the Association will be held on Thursday morning and on Friday morning and afternoon. It is expected that these Sessions will be devoted to the following topics: Professional Opportunities in Mathematics, Teacher Training, The School Mathematics Study Group.

NEWS ITEMS AND ANNOUNCEMENTS

POSTDOCTORAL RESEARCH ASSOCIATESHIPS IN MATHEMATICS OF THE OFFICE OF NAVAL RESEARCH. Under contract with various universities, the Office of Naval Research plans to continue its support of a number of postdoctoral research associateships in pure mathematics. The contracts will provide a salary of $6,000 for the academic year, an additional $1,300 for the summer, if desired, and an allowance of $500 for incidental expenses, including travel.

Appointments will be made on a competitive basis. Sole consideration will be given to the merits of the proposed research and the ability of the applicant as supported by letters of recommendation. Applications will be screened by the Office of Naval Research on the basis of an evaluation by the N. R. C. Committee on Mathematics, Advisory to the Office of Naval Research, consisting of L. Zippin (Chairman), R. P. Dilworth, G. A. Hedlund, M. Kac, I. Kaplansky, M. M. Schiffer, J. J. Stoker.

At the recommendation of the Committee, the following universities are being invited to participate in this program: Brown University, California Institute of Technology, Columbia University, Cornell University, University of Illinois, Johns Hopkins University, University of Michigan, Northwestern University, University of Notre Dame, Ohio State University, Purdue University, University of Virginia, University of Washington, Yale University.

Applications for the academic year 1959-60 must be submitted by 15 January 1959. Applicants will receive notification of the final decision about 15 April 1959. Application forms and further information may be obtained from Dr. Arthur Grad, Mathematics Branch, Office of Naval Research, Washington 25, D. C.

THE 1959 HEAT TRANSFER AND FLUID MECHANICS INSTITUTE will be held on June 11-13, 1959, at the University of California at Los Angeles. The objectives of the Institute will be to present technical and scientific advances in fluid mechanics, heat transfer, thermodynamics and related fields. Emphasis will be given to topics reflecting the problems of greatest current interest, such as low density flow, or radiative heat transfer of reacting gases, and also to topics in fields bridging the conventional divisions such as magneto-aerodynamics and aerothermochemistry.

The 1958 Institute was sponsored by the California Institute of Technology; Stanford University; University of California, Berkeley; University of California, Los Angeles; University of Santa Clara; University of Southern California; American Institute of Chemical Engineers; American Rocket Society; American Society of Mechanical Engineers; American Society of Refrigerating Engineers; and the Institute of the Aeronautical Sciences.

Papers of a fundamental nature in keeping with the above objectives are invited. Persons wishing to participate should notify the Papers Committee by providing a title and an abstract of their proposed papers not later than December 1, 1958. Authors will be notified of tentative acceptance by January 1, 1959. Completed papers must be submitted before February 15, 1959. Final acceptance will be based on a selective review of the completed papers only. All papers will be preprinted in full and the assembled sets will be on sale at the Institute meeting. Authors may feel free to submit their papers for subsequent publication to any journal of their own choosing.

The Papers Committee Co-Chairmen will be Artur Mager and Ernest Mayer. All correspondence regarding papers should be addressed to Mr. Mager, Astro-Marquardt Aircraft Company, P. O. Box 2013, South Annex, Van Nuys, California. For further information write to Eldon L. Knuth, General Chairman, 1959 Heat Transfer and Fluid Mechanics Institute, Institute of Industrial Cooperation, Department of Engineering, University of California, Los Angeles 24, California.

FOURTH CONGRESS ON THEORETICAL AND APPLIED MECHANICS OF THE INDIAN SOCIETY OF THEORETICAL AND APPLIED MECHANICS. The Congress will be held from December 28-31, 1958 at the Bengal Engineering College, Howrah (Calcutta), INDIA. The president will be Dr. S. R. Sen Gupta, M. I. E. Research papers may be contributed on any of the following topics: Elasticity-Plasticity-Rheology, Fluid Mechanics (Aerodynamics-Hydrodynamics), Mechanics of Solids (Ballistics-Vibrations-Friction-Lubrication), Statistical Mechanics-Thermodynamics-Heat Transfer, Mathematics of Physics and Mechanics-Methods of Com-

putation, and Experimental Techniques. A paper should reach the Secretary-Treasurer with three copies of its abstract before October 15, 1958.* There will be invited addresses of one-half hour each on special topics.

The registration fee for the Congress is Rs.10/-, which should be sent to the Secretary-Treasurer, Dr. B. R. Seth, Indian Institute of Technology, Kharagpur, INDIA. Board and lodgings arrangements for members and delegates will be made in the hostels of Bengal Engineering College, Howrah (Calcutta). It is expected that a railway fare concession in the form of a single fare for two journeys will be available for all members and delegates. Concession certificates will be issued in due course. Registration forms and any other information may be obtained from the Secretary-Treasurer, Dr. Seth, at the Indian Institute of Technology in Kharagpur.

* Editor's note: We regret that this item was received too late for insertion in the October NOTICES.

AN IN SERVICE TEACHERS INSTITUTE AT THE ILLINOIS INSTITUTE OF TECHNOLOGY, CHICAGO. This Institute is being held during the academic year 1958-59 on Saturday mornings. It will enable approximately 50 Chicago area public and private high school teachers to attend classes during the academic year and will be directed by Professor Haim Reingold, the Chairman of the Department of Mathematics, Illinois Institute of Technology. Professor Karl Menger will be one of the course lecturers. Basic concepts of mathematics will be stressed. The Institute was made possible by a National Science Foundation grant. Tuition for the course will be paid by the grant and a travel stipend will be paid to commuting teachers. Members of the Institute completing the program will be granted two semester hours of college-level credit. Further information may be obtained from Professor Haim Reingold, Chairman, Department of Mathematics, Illinois Institute of Technology, Technology Center, Chicago 16, Illinois.

A COMPUTING CENTER FOR THE UNIVERSITY OF KENTUCKY has been set up with an IBM 650 and peripheral equipment at the University in Lexington, Kentucky. It will be used for research and educational uses under the IBM Education Contribution Program. Dr. John W. Hamblen, formerly Associate Professor of Mathematics and Director of the Computing Center at Oklahoma State University, has been employed as full time Director of the Computing Center, which has been established under the general supervision of the Office of the Vice President. An advisory committee has been constituted with Dr. Leo M. Chamberlain, Vice President, as chairman. It in-

cludes various heads of departments and deans including Professor J. C. Eaves, the Head of the Department of Mathematics and Astronomy.

A CAREER BOOKLET CONCERNING MATHEMATICS entitled "Should You Be A Mathematician" has been issued by the New York Life Insurance Company. The article was originally addressed to parents and first appeared as a public service advertisement in The Saturday Evening Post and a number of other magazines. The Headquarters Offices of the Society have a supply of the booklets, and copies will be mailed to those requesting them as long as the supply lasts. Copies may also be obtained from the New York Life Insurance Company, Public Relations Department, 51 Madison Avenue, New York 10, New York.

DR. HENDRIK W. BODE BECOMES A VICE PRESIDENT OF BELL TELEPHONE LABORATORIES. Dr. Bode, who has been Director of Research in the Physical Sciences for the Bell Telephone Laboratories, was elected a Vice President as of October 1. Dr. Bode has been associated with Bell Telephone Laboratories since 1926. He will be in charge of one of the two vice presidential areas devoted to military development. He succeeds J. P. Molnar who was recently elected president of the Sandia Corporation and a vice president of the Western Electric Company. He is a member of the National Academy of Sciences.

A RESOLUTION ON BEHALF OF DR. LEON W. COHEN. At its meeting at the Massachusetts Institute of Technology on August 27 the Conference Board of the Mathematical Sciences, comprised of representatives from the American Mathematical Society, Mathematical Association of America, Institute for Mathematical Statistics, Association for Symbolic Logic, National Council of Teachers of Mathematics, and the Society for Industrial and Applied Mathematics adopted the following resolution:

Whereas, on July 15, 1958, Dr. Leon W. Cohen resigned from his position as Program Director for Mathematical Sciences of the National Science Foundation in order to resume work of an academic nature, and

Whereas, during his term of office in the National Science Foundation, there developed many serious problems for American mathematics, especially relating to the proper use of federal funds for research grants, summer institutes, publications, and the like, and

Whereas, Dr. Cohen handles these problems courageously,

impartially, and with wisdom, despite the difficulties perennially attendent on a position hemmed in on one side by the public and on the other by a governmental agency constantly accountable to the National Congress, and

Whereas, we feel that the entire field of mathematics has benefited from Dr. Cohen's efforts while Program Director,

Be it therefore resolved that this Board express its deep gratitude to Dr. Cohen for his services to mathematics during his incumbency, and

Be it further resolved that copies of this resolution be sent to Dr. Cohen, spread upon the minutes of this Board, and submitted to the member societies for publication as they deem fit.

A MESSAGE CONCERNING THE SCOPE OF THE PUBLICATIONS OF THE SOCIETY. The Secretary of the Society wishes to call attention to the fact that none of the Society's publications are restricted to particular branches of mathematics or to pure or to applied mathematics. In particular, the two series of books, the Colloquium Series and Surveys will accept manuscripts in applied as well as pure mathematics. Those having or expecting to have manuscripts in applied mathematics suitable for one or the other of these two series are invited to submit them to the Chairman of the appropriate editorial committee: Professor I. J. Schoenberg for the Surveys and Professor Salomon Bochner for the Colloquium.

PERSONAL ITEMS

(This section is restricted to members of the Society)

Northwestern University has received four grants from the National Science Foundation. The Directors of the projects will be: Professor Daniel Zelinsky, Assistant Professor Bruno Harris, Associate Professor M. A. Rosenlicht and Associate Professor Hsien-Chung Wang.

Associate Professor B. H. Bissinger, on leave from Lebanon Valley College, has been awarded a National Science Foundation Science Faculty Fellowship and will spend the year with the statistical research group at Princeton University.

Assistant Professor J. B. Chiccarelli of Fordham University has received a National Science Foundation Science Faculty Fellowship and will be at New York University for the academic year 1958-1959.

Assistant Professor F. W. Gehring, on leave from the University of Michigan for the next two years, will be at the University of Helsinki, Finland, for the academic year 1958-1959 and at the Swiss Federal Institute of Technology, Zurich, Switzerland for the academic year 1959-1960. He will be supported during these two years by Fellowships from the Guggenheim Foundation and from the National Science Foundation and by a Fulbright Fellowship from the State Department.

Dr. Baruch Germansky of Jerusalem, Israel has received a scholarship from the Technische Universität, Berlin, Germany which will enable him to be there for one year.

Assistant Professor W. M. Gilbert, on leave from Iowa State College of Agriculture and Mechanic Arts to accept a National Science Foundation Visiting Fellowship at Princeton University, has been promoted to an associate professorship.

Dr. M. W. Hirsch of the University of Chicago has received a National Science Foundation Post-Doctoral Fellowship and will be at the Institute for Advanced Study for the academic year 1958-1959.

Associate Professor L. E. Payne, on leave from the University of Maryland, has been awarded a National Science Foundation Senior Post Doctoral Fellowship and will be at Kings College, New Castle-upon-Tyne, England, for the academic year 1958-1959.

Professor I. E. Segal of the University of Chicago has been awarded a National Science Foundation Senior Postdoctoral Fellowship and will be at the University of Copenhagen, Denmark, for the academic year 1958-1959.

Assistant Professor R. V. Chacon, Dr. Joshua Chover, Professor P. C. Hammer, Professor C. C. MacDuffee, and Professor W. R. Wasow, all members of the Mathematics Department, hold part-

time appointments at the Mathematics Research Center of the University of Wisconsin, Madison. Assistant Professor M. W. Johnson, Jr. divides his time between the Research Center and the Mechanics Department.

Mr. R. O. Abernathy of the University of California, Berkeley, has been appointed to an assistant professorship at Southern University and Agricultural and Mechanical College.

Dr. W. A. Al-Salam of Duke University has been appointed to an assistant professorship at the University of Baghdad, Iraq.

Dr. H. A. Antosiewicz of the National Bureau of Standards, Washington, D. C., has been appointed to a visiting associate professorship at the University of Southern California.

Professor R. F. Bell, on leave from Eastern Washington College of Education, has been appointed a lecturer at the University of Michigan.

Dr. B. P. Bogert of the Bell Telephone Laboratories, Inc. has accepted a position as head of the acoustics department of the Bendix Aviation Corporation, Southfield, Michigan.

Mr. J. R. Boyd of Chance-Vought Aircraft, Inc. has been appointed to an assistant professorship at Lamar State College of Technology.

Dr. F. G. Brauer of the University of Chicago has been appointed a lecturer at the University of British Columbia.

Dr. E. H. Brown, Jr. of Brown University has been appointed to an assistant professorship at Brandeis University.

Dr. R. W. Brown of Oregon State College has accepted a position as research analyst with Boeing Airplane Company, Seattle, Washington.

Professor Herbert Busemann, on leave from the University of Southern California, has been appointed to a visiting professorship at Harvard University.

Assistant Professor R. K. Butz of Colorado State University has been appointed to an associate professorship at Alabama Polytechnic Institute.

Mr. B. R. Buzby of Indiana University has accepted a position as research mathematician with the Electro Metallurgical Company, Niagara Falls, New York.

Professor D. G. Chapman, on leave from the University of Washington, has been appointed to a visiting professorship at North Carolina State College.

Dr. K.-T. Chen of the University of Hong Kong has been appointed to an associate professorship at the Instituto Tecnologico de Aeronautica, Saõ Paulo, Brazil.

Mr. B. F. Cheydleur of Sperry Rand Corporation has accepted a position as advanced programming manager with the Radio Corporation of America, Camden, New Jersey.

Associate Professor P. G. Comba will be on leave from the University of Hawaii and will be at the Western Data Processing Center of the University of California, Los Angeles, for the academic year 1958-1959.

Associate Professor Byron Cosby, Jr. of the State University of Iowa has been appointed to a professorship at the University of Texas.

Associate Professor C. H. Cunkle of Dickinson College has accepted a position as research mathematician with Cornell Aeronautical Laboratories, Inc., Buffalo, New York.

Associate Professor C. W. Curtis, on leave from the University of Wisconsin, has been appointed to a visiting associate professorship at Cornell University.

Professor Harold Davenport of the University of London, England, has been appointed to a professorship at Trinity College, Cambridge University, Cambridge, England.

Dr. R. T. Dillon of the University of Oregon has accepted a position as staff member with Sandia Corporation, Albuquerque, New Mexico.

Professor R. J. Duffin, on leave from Carnegie Institute of Technology, has been appointed director of special research in applied mathematics at Duke University.

Dr. A. V. Fend of Technical Operations, Inc. has accepted a position as mathematical statistician with the Stanford Research Institute, Menlo Park, California.

Associate Professor M. K. Fort, Jr. of the University of Georgia has been awarded an Alfred P. Sloan Fellowship for the years 1958-59 and 1959-60.

Professor José Gallego-Diaz of the University of Madrid, Spain, has been appointed to a visiting professorship at the University of Puerto Rico, Rio Piedras, Puerto Rico.

Dr. L. D. Gates, Jr. of the Naval Proving Ground, Dahlgren, Virginia has accepted a position as mathematician with the Babcock and Wilcox Company, Lynchburg, Virginia.

Dr. Abolghassem Ghaffari of the National Bureau of Standards, Washington, D. C. has joined the staff of the Mathematics Research Center of the University of Wisconsin, Madison.

Associate Professor Simon Green of the University of Tulsa has been appointed to an associate professorship at the University of South Carolina.

Dr. John Greever of the University of Virginia has been appointed to an assistant professorship at Florida State University.

Miss Louisa S. Grinstein of Hunter College has accepted a position as systems analysis engineer with Republic Aviation Corporation, Mineola, New York.

Dr. J. K. Hale of Remington Rand UNIVAC has accepted a position as mathematician with RIAS, Inc., The Martin Company, Baltimore, Maryland.

Assistant Professor J. W. Hamblen of Oklahoma State University has been appointed director of the computing center and associate professor of statistics at the University of Kentucky.

Dr. D. K. Harrison of Brown University has been appointed to an assistant professorship at Haverford College.

Dr. G. M. Helmberg of the University of Washington has been appointed to an assistant professorship at Tulane University of Louisiana.

Professor T. H. Hildebrandt, on leave from the University of Michigan, has begun a three month appointment to the staff of the Mathematics Research Center of the University of Wisconsin, Madison.

Dr. F. C. Huckemann of Harvard University has been appointed to a position as privatdozent at the University of Giessen, Giessen-Lahn, Germany.

Associate Professor T. E. Hull of the University of British Columbia has been appointed to a visiting associate professorship at the California Institute of Technology.

Assistant Professor Jack Indritz of Washington University has been appointed to a visiting assistant professorship at the University of Minnesota.

Dr. R. T. Ives of the University of Virginia has been appointed to an assistant professorship at Harvey Mudd College.

Dr. A. T. James of the University of Adelaide, Australia has been appointed a visiting lecturer at Yale University.

Dr. T. R. Jenkins of Lockheed Aircraft Corporation has been appointed to an assistant professorship at the State College of Washington.

Dr. J. G. Jewell of the Martin Company has accepted a position as special projects engineer with E. I. Du Pont de Nemours and Company, Newark, Delaware.

Assistant Professor J. B. Johnston of the University of Kansas City has been appointed to an assistant professorship at the University of Kansas.

Assistant Professor D. A. Kearns of the University of Maine has been appointed to a professorship at Merrimack College.

Assistant Professor R. P. Kelisky of the University of Texas has accepted a position as associate mathematician with the International Business Machines Corporation, Yorktown Heights, New York.

Mr. J. H. Keller of the University of North Carolina has been appointed to an assistant professorship at Furman University.

Associate Professor O. M. Klose of Seattle University has been appointed to an associate professorship at Humboldt State College.

Dr. J. K. Knowles of Massachusetts Institute of Technology has been appointed to an assistant professorship at California Institute of Technology.

Mr. S. B. Kochen of Princeton University has been appointed to an assistant professorship at the University of Montreal.

Professor John Kronsbein of Evansville College has been appointed to a professorship at the University of Florida.

Dr. J. E. LeBel of Montreal, Canada has been appointed to an assistant professorship at Georgetown University.

Dr. Milton Lees of the Shell Development Company has been appointed a temporary member of the Institute for Advanced Study.

Professor G. G. Lorentz of Wayne State University has been appointed to a professorship at Syracuse University.

Dr. D. B. Lowdenslager of the University of Illinois has been appointed a lecturer at Princeton University.

Mr. W. M. Lowney of the University of Notre Dame has accepted a position as scientist with the Missile Systems Division, Lockheed Aircraft Corporation, Palo Alto, California.

Assistant Professor W. A. J. Luxemburg of the University of Toronto has been appointed to an assistant professorship at the California Institute of Technology.

Mr. B. L. McAllister of the University of Wisconsin has been appointed to an assistant professorship at South Dakota School of Mines and Technology.

Dr. M. D. McIlroy of Massachusetts Institute of Technology has accepted a position as research mathematician with the Bell Telephone Laboratories, Inc., Murray Hill, New Jersey.

Dr. A. W. McKinney, III of the University of California, Berkeley, and the California Research Corporation has accepted a position as mathematician in the Vallecitos Atomic Laboratory of the General Electric Company, Pleasanton, California.

Mr. C. W. Marshall has accepted a position as applied mathematician with the Institute for Defense Analyses, Washington, D. C.

Associate Professor E. P. Miles, Jr. of Alabama Polytechnic Institute has been appointed to an associate professorship at Florida State University.

Assistant Professor Marian A. Moore, on leave from Southern Illinois University, has been appointed to a visiting professorship at Hiram College.

Dr. G. W. Morgenthaler of the Institute for Air Weapons Research, University of Chicago, has been appointed to an associate professorship in the undergraduate division of the University of Illinois, Chicago.

Dr. August Newlander, Jr. of the University of Washington has been appointed a research associate at the University of Chicago.

Assistant Professor Katsumi Nomizu of Nagoya University, Japan, has been appointed to an associate professorship at the Catholic University of America.

Dr. Edward Norman of Cornell University has been appointed to an assistant professorship at Michigan State University.

Dr. A. B. Novikoff of the University of California has accepted a position as research mathematician with the Stanford Research Institute, Menlo Park, California.

Dr. Alexander Orden of Burroughs Corporation has been appointed to a professorship in the School of Business, University of Chicago.

Dr. S. V. Parter of Brookhaven National Laboratory has been appointed assistant professor and assistant director of the Computing Center at Indiana University.

Mr. C. R. Paulson of ACF Industries, Inc., has accepted a position as project engineer and coordinator with Link Aviation, Inc., Binghamton, New York.

Professor M. M. Peixoto, on leave from the University of Brazil, has accepted a position as staff scientist with RIAS, Inc., The Martin Company, Baltimore, Maryland.

Dr. R. R. Phelps of the University of Washington has been appointed a member of the Institute for Advanced Study for two years.

Associate Professor Philburn Ratoosh, on leave from the Psychology Department of Ohio State University, has been appointed to a visiting associate professorship in the Psychology Department of the University of California, Berkeley.

Professor Emeritus H. W. Reddick of New York University has been appointed to a visiting professorship at Syracuse University.

Professor R. F. Rinehart, on leave from Case Institute of Technology, will be director of special research and operations research at Duke University.

Professor C. A. Rogers of the University of Birmingham, England, has been appointed to a professorship at the University of London, England.

Mr. Eugene Rogot of Syracuse University has accepted a position as analytical statistician with the National Institutes of Health, Bethesda, Maryland.

Associate Professor Louise J. Rosenbaum, on leave from Saint Joseph College, Connecticut, will be in Oxford, England for the academic year 1958-1959.

Professor R. A. Rosenbaum, on leave from Wesleyan University, has been awarded a National Science Foundation Science Faculty Fellowship and will be at Oxford University, England for the academic year 1958-1959.

Professor S. A. Rowland, retired from Ohio Wesleyan University with the title Professor Emeritus, has been appointed a lecturer at Ohio State University.

Mr. E. D. Schell of the Sperry Rand Corporation has accepted a position as advisory mathematician with the International Business Machines Corporation, Yorktown Heights, New York.

Assistant Professor L. A. Schmittroth of Montana State University has accepted a position as supervisor of the applied mathematics section, atomic energy division, Phillips Petroleum Company, Idaho Falls, Idaho.

Mr. T. I. Seidman of New York University has been appointed a mathematician in the Radiation Laboratory, University of California, Livermore.

Professor S. M. Shah of Muslim University, Aligarh, India has been appointed to a visiting professorship at the University of Wisconsin, Madison, for the period September 1958-January 1959.

Assistant Professor L. J. Snell of Dartmouth College has been promoted to an associate professorship and will be on leave at Stanford University for the academic year 1958-1959 to accept a National Science Foundation Fellowship.

Dr. Johann Sonner of Wright Patterson Air Force Base, Ohio has been appointed to an associate professorship at the University of South Carolina.

Assistant Professor F. L. Spitzer of the California Institute of Technology has been appointed to an associate professorship at the University of Minnesota.

Dr. J. H. Stapleton of the General Electric Company has been appointed to an assistant professorship at Michigan State University of Agriculture and Applied Science.

Professor R. R. Stoll, on sabbatical leave from Oberlin College, will be a senior research fellow at California Institute of Technology for the academic year 1958-1959.

Dr. D. D. Strebe has returned to the staff of the University of South Carolina from the Teachers College, State University of New York, Oswego, as Associate Professor of Mathematics.

Dr. K. R. Stromberg of the University of Washington has been appointed a research associate at Yale University.

Dr. O. E. Taulbee of the Sperry Rand Corporation, Saint Paul, Minnesota has accepted a position as mathematician with Lockheed Aircraft Corporation, Marietta, Georgia.

Assistant Professor G. L. Thompson of Dartmouth College has been appointed to a professorship at Ohio Wesleyan University.

Assistant Professor E. H. Tompkins, Jr. of North Carolina State College has accepted a position as member of the technical staff of the Space Technology Laboratories division of the Ramo-Wooldridge Corporation, Los Angeles, California.

Dr. R. N. Townsend of the University of Illinois has been appointed to an assistant professorship at San Jose State College.

Mr. D. A. Trumpler of Massachusetts Institute of Technology has been appointed to an assistant professorship at Georgia Institute of Technology.

Professor S. E. Warschawski, on leave from the University of Minnesota, has been appointed to a visiting professorship at the University of California, Los Angeles.

Dr. Edwin Weiss of Harvard University has been appointed to an assistant professorship at the University of California, Los Angeles.

Dr. Mary B. Weiss of the University of Chicago has been appointed to an assistant professorship at De Paul University.

Assistant Professor C. H. Wilcox, on leave from the California Institute of Technology, will be a visiting member of the Mathematics Research Center, University of Wisconsin, Madison, for the academic year 1958-1959.

Dr. R. E. Wild of the Douglas Aircraft Corporation has been appointed to an assistant professorship at the University of Arizona.

Dr. J. H. Williamson of the Queen's University, Belfast, Northern Ireland, has been appointed a lecturer at Cambridge University, Cambridge, England.

Dr. G. M. Wing of the University of California, Los Alamos, New Mexico, has been appointed to an associate professorship at the University of New Mexico.

Professor Emeritus Roscoe Woods of the State University of Iowa has been appointed to a visiting professorship at Grinnell College for the academic year 1958-1959.

Dr. Kapbyung Yoon of Syracuse University has been appointed to an assistant professorship at Seoul University, Seoul, Korea.

Assistant Professor E. M. Zaustinsky of San Jose State College has been appointed to an assistant professorship at Santa Barbara College, University of California.

Dr. Mishael Zedek of the University of California has been appointed to an assistant professorship at the University of Maryland.

The following promotions are announced:

A. G. Azpeitia, University of Massachusetts, to an assistant professorship.

Mr. C. M. Bruen, International Business Machines Corporation, Endicott, New York, to an associate mathematician.

Dr. S. D. Conte, Ramo-Wooldridge Corporation, to manager of the mathematical analysis department, space technology laboratories.

M. L. Curtis, University of Georgia, to a professorship.

A. L. Fass, Queens College, to an assistant professorship.

W. H. Fuchs, Cornell University, to a professorship.

B. A. Galler, University of Michigan, to an assistant professorship.

John Gurland, Iowa State College of Agriculture and Mechanic Arts, to a professorship.

A. B. Lehman, Case Institute of Technology, to an assistant professorship.

E. K. Rabe, University of Cincinnati, to a professorship.

W. A. Small, Grinnell College, to an associate professorship.

J. J. Stoker, New York University, to director of the Institute of Mathematical Sciences.

S. I. Vrooman, University of Pittsburgh, to an associate professorship.

The following appointments to instructorships are announced:

Brooklyn College: Dr. Silvio Aurora; University of California, Los Angeles: Dr. E. O. Thorp, Dr. H. J. Weinitschke; University of California, Riverside: Dr. R. L. McKinney; Columbia University: Dr. Elliott Mendelson; Corning Community College: Mr. H. I. Gross; Dartmouth College: Dr. B. H. Sams; DePaul University: Dr. J. G. Thompson; Northwestern University: Mr. W. J. Cody, Jr.; Princeton University: Dr. H. H. Johnson, Mr. W. T. Sharp; University of Rochester: Dr. D. W. Dean, Dr. N. J. Rothman; Rutgers, The State University: Mr. S. A. Foote, Mr. W. R. Jones; Trinity College, Hartford, Connecticut: Dr. M. J. Poliferno; Vassar College: Miss Edith Moss; Virginia State College: Mrs. Emma D. B. Smith; Western Reserve University: Mrs. Ruth D. O'Dell.

Correction:

Dr. P. M. Whitman of the applied physics laboratory of Johns Hopkins University, Silver Spring, has been awarded a William S. Parsons Fellowship at Johns Hopkins University, Baltimore, Maryland for the academic year 1958-1959. He was incorrectly listed as P. M. Parsons in the October issue of the NOTICES.

LETTERS TO THE EDITOR

Editor's Note: In the future, authors of letters to be published in this department of the NOTICES are asked please to limit themselves to 1,000 words. Longer letters will be published only by direction of the Council.

Editor, the NOTICES.

The central purpose of this letter is to indicate some of the potentialities of mathematical reviewing when it is interpreted as work which can unify the whole everexpanding mathematical research literature. In effect, the aim of this generalized form of mathematical reviewing journalism should be to collect continually all the published mathematical research which is being scattered over the surface of the earth throughout some one thousand journals; to endow this whole immense output with an informing order and organic unity; and to nourish research labours by imparting to the mathematicians a keen awareness of the existence of new mathematical results and a clear global view of their literature. Every mathematical reviewing journal has the following two functions: reviewing in the small and reviewing in the large. We discuss each of these, and carefully interlace them with our ideas.

REVIEWING IN THE SMALL. Let us first examine the basic meaningful element of a mathematical reviewing journal, i.e., the review article of a research paper. (A good research paper, according to Saunders MacLane, presents a promising new idea when it is hot -- and when nobody knows for sure that it will turn out to be really useful.) The principal purpose of a review article should be to summarize as scrupulously as possible the main content of a research paper and not just haphazardly describe the work. In truth, a descriptive review does not satisfy the minimum requirements of an informing and complete review. Indeed, the research reader does not learn much from a descriptive review. The following whole review illustrates a typical descriptive review: "A theorem is proved which is somewhat more general than the following corollary. Let f be a continuous, single-valued function in a region R of the complex plane and let $\{U_i\}$ be an arbitrary open covering of R. If $\int_C f(z)dz = 0$ for every circle C that lies entirely within at least one open set of the covering $\{U_i\}$, then f is holomorphic in R." Another common type of descriptive review consists of an enumeration of the section headings of a research paper, and concludes with something like "The paper is long treatment on the subject, weighs six ounces on the kitchen scales." More sophisticated descriptive reviews inevitably leave the readers with questions: If "the author proves a number of important theorems" then precisely what is at least the principal theorem of the research

paper; if "the author gives an ingenious proof", then roughly what is the gist of the proof; if "the author presents a rather simple proof of this well-known theorem", then what is the principal tool employed in the proof? And furthermore, many times an ingenious employment of powerful methods will give a proof a very simple appearance, hence the research paper is all the more important and should not be given the illusion of being trivial. In such a descriptive contingency, a review article does not contribute to the needs of the research workers at all, and may very well conceal an important mathematical result. On the other hand, a review article should not be a substitute for the reading of the original paper.

It is very useful to define the basic structure of a complete and informing review article in the following manner. In written prose a complete sentence consists of a subject (S) and a predicate (P). Analogously, a complete review could be defined as a sort of sentence. It should also consist of an (S) and a (P). The (S) of a review article should consist of a statement of the main result of the original paper, and whatever else that may be necessary to complete the sense of the main result, such as the definitions of the concepts employed and the necessary bibliographical references. The (P) should consist of an indication of the proof or development of the main result, and other necessary qualifying statements needed to complete a review.

REVIEWING IN THE LARGE. One review article of a researcher's work is but a part of a reviewing unit; its conclusive significance should come from its relation to the sets of reviews making up the monthly and annual reviewing compendiums. The three necessary reviewing-in-the-large tasks are (1) the classification of sets of review articles comprising the standard monthly issues, (2) the index of a mathematical reviewing journal for a given volume, and (3) the global review of the current mathematical research literature for a given year.

(1) Classification. A monthly issue of review articles of the published mathematical research literature may be classified from the following two points of view.

The first, the most popular with mathematical reviewing journals, consists of pouring all the review articles of the whole ever-expanding research literature into a preconceived system of extremely rigid and detailed compartments of Algebra, of Analysis, and of Geometry. In truth, they are not classifying the reviews of the current literature, but seem to be constructing a Procrustean classification of mathematics, and then unfortunately pounding all the review articles of published mathematical labours into it. In this age where research labours perpetually keep unfolding new and sovereign theories of mathematics, it appears strange that mathematical reviewing journals should still insist, in their classifying efforts, on capturing these majestic mathematical labours into the dusty vaults of Algebra,

Analysis and Geometry. As a result, for example, the reviews of work on Lie theory and homological algebra, in spite of the beauty and importance of these theories, are still being beaten into several different abodes. Hence the readers are obliged very frequently to guess unnecessarily at whether a particular review article is misclassified or if the original paper contains some overlooked material justifying the assigned classification. Surely, from the standpoint of the readers, review articles that are dispersed in such misapplied orders and finally concealed under unrelated general headings cannot be conveyed to another's mind without loss and hardly likely to carry much meaning to the readers at all. Unfortunately, this approach has no concern for the growing modern tendencies of mathematics, and utterly fails to anticipate and adjust itself to the momentum of mathematical research.

The second approach fully takes into consideration the growing tendencies and momentum of mathematical research progress. It is achieved by granting sovereignty to each mathematical theory. Review articles of mathematical research papers would be ordered according to the theories to which they pertain. Reviews of papers on a new theory T which draws its origin from theory T' and its methods from still another theory T" need not be classified under either or both T' and T" but more naturally under theory T itself. For example, review articles on homological algebra, Lie theory, theory of manifolds, theory of algebraic varieties, theory of class formations, theory of distributions and so on would be found in sufficient numbers under their own respective headings rather than being crudely dispersed under several unrelated headings. If the review articles are of the (S)-(P) type mentioned previously, then the (S)'s of our reviews can serve to determine the appropriate theories to which the review articles belong. Furthermore, all the mathematical tools mentioned in the (P)'s of our reviews can be cross-referenced to their appropriate theories. These cross-references would clarify theories, and would indicate their close connections with other theories that touch them directly as well as their more remote connections with still other theories that might otherwise be thought irrelevant.

We will now give a sample of our fluid classification of review articles as it would look in the Table of Contents of a mathematical reviewing journal using our reviewing methods (this Table of Contents would depend on the subject matter on hand, hence it would differ with each issue):

THEORIES

Theory of sets; Theories of algebraic systems; Homological algebra; Theory of algebraic varieties; Theory of class formations; Theory of algebraic numbers;

Theories of topological systems; Theory of topological groups; Theory of manifolds; Lie Theory; Algebraic topology; Algebraic geometry; Differential geometry; Theory of integration; Theory of distributions; measure theory; Real function theory; Complex function theory; Theory of ordinary differential equations; Theory of partial differential equations; Theory of Probability.

MATHEMATICAL METHODS IN THE SCIENCES

Computational sciences; Physical science; Engineering sciences; Biological sciences; Social sciences.

In view of the fact that there exist reviewing journals for the various scientists and technologists, the costly duplication of reviewing extensively their sciences in the mathematical reviewing journals should be avoided. It is sufficient simply to review those papers which directly apply mathematical methods to scientific situations, e.g., theory of groups, theory of differential equations, theory of distributions, etc., to Physics, etc.

(2) Index. The value of a compendium such as we have defined can be seriously reduced by an inadequate index. An index should serve as a signal to the highly valuable elements contained in a set of individual review articles for a given year. Mathematical reviewing journals, in general, have managed for years to lose valuable definitions, with bibliographical references, of new concepts and ideas discussed in reviews, because of their inadequate indexes. Moreover, this type of explicit indexing would certainly go a long way toward improving mathematical nomenclature. Such an index, with a little ingenuity, can be easily compiled from the manuscripts of reviews by standard manual techniques. Furthermore, there are techniques for the rapid compilation of analytical indexes using either conventional punched-card systems or electronic data-processing machines. A detailed description of the procedures used in automatically indexing scholarly work is given by P. Tasman (IBM J. Res. Develop., vol. 1, 1957, pp. 249-256). Incidentally, in view of the fact that for each review article contained in a mathematical reviewing journal there corresponds about a half dozen office record cards which are shuffled and reshuffled dozens of times before a review appears in print, the introduction of data-processing tools in reviewing work would indeed be a significant factor in accelerating and improving the reviewing services. (There exist indications to support the fact that the Bureau of Standards is interested in this assignment.)

(3) Global Review. In reviewing in the small, our exclusive aim should be to summarize as scrupulously as possible the principal

content of an individual mathematical work. In turn, global reviewing unfolds a task of more formidable scope: A global review of the whole current mathematical research literature. This aspect of reviewing may eventually provide the mathematical community with a much needed over-all picture of their current research labours. It may be accomplished by treating the contents of annual collections of review articles of the (S)-(P) type discussed above. This undertaking would require a "circle of reviewers" rather than an individual reviewer, i.e., it would have to be a corporate enterprise in the sense of, say, Bourbaki. A global review article of this kind could be issued together with the conventional annual index. Such a culminating form of reviewing would certainly help to elicit more fertile contributions from the individual reviewers, and a more discreet distribution of the reviewing raw materials from the editors. Moreover, it would indicate to the mathematical community the high-quality mathematical work which is generally, as André Weil points out, diluted with more modest material. Perhaps, it would also clearly unfold to the mathematical hierarchy the need for locating their reviewing journals in high-powered mathematical centers, e.g., Princeton, New Jersey.

We offer this idea of mathematical reviewing journalism to our fellow-members of the Society for further scrutiny and, we hope, development.

H. A. Pogorzelski

Editor's Note: The above letter, written by an Editorial Associate on the Mathematical Reviews staff, is published with the consent of the Chairman of the Mathematical Reviews Editorial Committee and the Executive Editor of Mathematical Reviews.

NEW PUBLICATIONS

do we have I-IV? yes

Advances in applied mechanics. Vol. V. New York, Academic Press, 1958. 10 + 459 pp. $12.00.

Aleksandrov, P. S. Topologia combinatoria. (Manuali: serie di matematica.) Turin, Einaudi, 1957. 18 + 750 pp. 10,000 Lire.

Alexandrow, A. D. Konvexe polyeder. (Mathematische Lehrbücher und Monographien. Ed. by the Deutschen Akademie der Wissenschaften zu Berlin, Forschungsinstitut für Mathematik. Part II: Mathematische Monographien, vol. 8.) Berlin, Akademie-Verlag, 1958. 10 + 419 pp. 37.00 DM.

Archbold, J. W. Algebra. London, Pitman, 1958. 19 + 440 pp. 45s.

Arsham, I. See Salzer, H. E.

Aubert, M. See de Beaumont, H. du B.

Bartel, K. Geometria wykreślna. [Descriptive geometry.] 4th ed. Warsaw, Państwowe Wydawnictwo Naukowe, 1958. 427 pp.

Bateman, P. T. See Landau, E.

Beach, E. F. Economic models: an exposition. New York, Wiley, 1957. 11 + 227 pp. $7.50.

de Beaumont, H. du B. Étude critique de certains prolongements de la mécanique rationnelle: thermodynamique et viscosité. Preface by M. Aubert. (Publ. Sci. Tech. Ministère de l'Air, Notes Tech. no. 72.) Paris, 1957. 8 + 47 pp. 1000 fr.

Bell, J. S. See Landau, L. D.

Bellman, R. E., Glicksberg, I., and Gross, O. A. Some aspects of the mathematical theory of control processes. (Rand Corporation Report No. R-313.) Santa Monica, Rand Corporation, 1958. 19 + 244 pp.

Bernays, P. See Wittenberg, A. I.

Bianchi, L. Opere. Ed. by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Vol. 9. Geometria degli spazi di Riemann. 264 pp. Vol. 10. Ricerche varie. 79 pp. Bound as one volume. Rome, Cremonese, 1958. 3500 Lire.

Blom, G. Statistical estimates and transformed beta-variables. Stockholm, Almqvist and Wiksells, 1958. 174 pp.

Boll, R. H., Leacock, J. A., Clark, G. C., and Churchill, S. W. Tables of light-scattering functions: relative indices of less than unity, and infinity. Ann Arbor, University of Michigan Press, 1958. 8 + 360 pp. $9.50.

Boole, G. Calculus of finite differences. Ed. by J. F. Moulton. 4th ed. New York, Chelsea, 1957. 12 + 336 pp. $4.95.

Bronstein, I. N., and Semendjajew, K. A. Taschenbuch der Mathematik: für Ingenieure und Studenten der technischen Hochschulen. Leipzig, Teubner, 1958. 12 + 548 pp., 1 insert. 22.50 DM.

Brun, V. See Netto, E.

Carrière, P. Méthodes théoriques d'étude des écoulements supersoniques. Preface by G. Darrieus. (Publ. Sci. Tech. Ministère de l'Air, no. 339.) Paris, 1957. 11 + 234 pp. 3000 fr.

Čebotarev, A. S. Sposob naimen'ših kvadratov s osnovami teorii veroyatnostei. Moscow, Izdat. Geodez. Lit., 1958. 606 pp., 1 insert. 14.50 rubles.

Chapin, N. An introduction to automatic computers. Princeton, Van Nostrand, 1957. 8 + 525 pp., 1 plate. $8.75.

Chew, V. See Experimental designs in industry.

Churchill, R. V. Operational mathematics. 2d ed. New York, McGraw-Hill, 1958. 9 + 337 pp. $7.00.

Churchill, S. W. See Boll, R. H.

Clark, G. C. See Boll, R. H.

A comprehensive bibliography on operations research through 1956, with supplement for 1957. (Publications in Operations Research, No. 4.) New York, Wiley, 1958. 11 + 188 pp. $6.50.

Cowling, T. G. Magnetohydrodynamics. (Interscience Tracts on Physics and Astronomy, No. 4.) New York, Interscience, 1957. 8 + 115 pp. Paperbound, $1.75; clothbound, $3.50.

Darrieus, G. See Carrière, P.

Davis, M. Computability and unsolvability. (McGraw-Hill Series in Information Processing and Computers.) New York, McGraw-Hill, 1958. 25 + 210 pp. $7.50.

Dobrowolski, W. W. Theorie der Mechanismen zur Konstruktion ebener Kurven. Berlin, Akademie Verlag, 1957. 8 + 134 pp. 18.00 DM.

Eagle, A. The elliptic functions as they should be: an account, with applications, of the functions in a new canonical form. Cambridge, Galloway and Porter, 1958. 28 + 510 pp. 45s.

Eichelbrenner, E. A. Décollement laminaire en trois dimensions sur un obstacle fini. (Office National d'Etudes et de Recherches Aéronautiques, Publ. No. 89.) 1957. 37 pp.

Eirich, F. R. See Rheology.

Elias, P. List decoding for noisy channels. (Research Laboratory of Electronics, Massachusetts Institute of Technology, Rep. No. 335.) Cambridge, 1957. 12 pp.

Emmons, H. W. See Fundamentals of Gas Dynamics.

Experimental designs in industry. A symposium held November 5-9, 1956 at North Carolina State College. Ed. by V. Chew. (Wiley Publications in Applied Statistics.) New York, Wiley, 1958. 11 + 268 pp.

Feinstein, A. Foundations of information theory. (McGraw-Hill Electrical and Electronic Engineering Series.) New York, McGraw-Hill, 1958. 10 + 137 pp. $6.50.

Flügge, S. See Handbuch der Physik.

Flügge, W. Statik und Dynamik der Schalen. 2d, rev. ed. Berlin, Springer, 1957. 8 + 286 pp. 28.50 DM.

Franklin, P. Functions of complex variables. (Prentice-Hall Mathematics Series.) Englewood Cliffs, Prentice-Hall, 1958. 9 + 246 pp. $6.95.

Fundamentals of Gas Dynamics. Vol. 3. High Speed Aerodynamics and Jet Propulsion. Ed. by H. W. Emmons. Princeton University Press, 1958. 13 + 749 pp. $20.00.

Gass, S. I. See Riley, V.

Gheorghiță, St. I. Capitole din teoria mișcărilor în mediile poroase. [Chapters in the theory of motion in porous media.] (Academia Republicii Populare Romine, Știință și Tehnică, No. 11.) Bucharest, Editura Tehnică, 1957. 116 pp., 1 insert. 3.25 Lei.

Glicksberg, I. See Bellman, R. E.

Goldberg, S. Introduction to difference equations, with illustrative examples from economics, psychology, and sociology. New York, Wiley, 1958. 12 + 260 pp. $6.75.

Goodell, J. D. The world of ki. St. Paul, Riverside Research Press, 1958. 4 + 215 pp. Paperbound, $3.00; clothbound, $4.50.

Goodman, J. E. See Landau, E.

Goodman, N. R. On the joint estimation of the spectra, cospectrum and quadrature spectrum of a two-dimensional stationary Gaussian process. (Engineering Statistics Laboratory, College of Engineering, New York University, Scientific paper No. 10, Nobs-72018 (1734-F) and Nonr-285 (17).) New York, 1957. 5 + 168 pp.

Gross, O. A. See Bellman, R. E.

Guderley, K. G. Theorie schallnaher Strömungen. Berlin, Springer, 1957. 15 + 376 pp. 42.00 DM.

Günther, W. See Weber, C.

Guter, R. S. See Kačmaž, S.

Handbuch der Physik. Ed. by S. Flügge. Vol. 6. Elastizität und Plastizität. Berlin, Springer, 1958. 145.00 DM.

Hardy, G. H., and Wright, E. M. Einführung in die Zahlentheorie. Trans. by H. Ruoff. Munich, Oldenbourg, 1958. 16 + 480 pp. 74.00 DM.

Hazelwood, R. N. See Peck, L. G.

Heber, G., and Weber, G. Grundlagen der modernen Quantenphysik. Part 2. Quantenfeldtheorie. (Mathematisch-Naturwissenschaftliche Bibliothek, Vol. 2.) Leipzig, Teubner, 1957. 6 + 138 pp. 7.50 DM.

Heffter, L. Grundlagen und analytischer Aufbau der Projektiven, Euklidischen, Nichteuklidischen Geometrie. 3d, substantially rev. ed. Stuttgart, Teubner, 1958. 191 pp. 16.80 DM.

Hil'mi, G. F. Kačestvennye metody v probleme n tel. Moscow, Izdat. Akad. Nauk SSSR, 1958. 123 pp., 1 insert. 2.95 rubles.

Hirzebruch, F., and Scheja, G. Garben- und Cohomologietheorie. Vol. 22. Munster, Aschendorff, 1957. 4 + 236 pp. 20.00 DM.

Holtmann, F. Mathematik. Vol. 1. Arithmetik. 4th, rev. ed. Leipzig, Fachbuchverlag, 1958. 14 + 322 pp. 6.00 DM.

James, G. See The tree of mathematics.

Kačmaž, S., and Šteĭngauz, G. Teoriya ortogonal'nyh ryadov. Trans. with a summary of later results by R. S. Guter and P. L. Ul'yanov. Ed., with supplements, by N. Ya. Vilenkin. Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1958. 507 pp., 1 insert. 17.50 rubles.

Kaplansky, I. An introduction to differential algebra. (Actualités Sci. Ind., No. 1251; Publ. Inst. Math. Univ. Nancago, No. 5.) Paris, Hermann, 1957. 63 pp. 800 fr.

Kohlbecker, E. E. See Landau, E.

Kollbrunner, C. F., and Meister, M. Ausbeulen: Theorie und Berechnung von Blechen. Berlin, Springer, 1958. 11 + 344 pp. 42.00 DM.

Krygowska, Z. Konstrukcje geometryczne na płaszczyźnie. [Geometrical constructions in the plane.] Warsaw, Państwowe Wydawnictwo Naukowe, 1958. 196 pp., 1 insert. 15 zł.

Landau, E. Elementary number theory. Trans. by J. E. Goodman. With exercises by P. T. Bateman and E. E. Kohlbecker. New York, Chelsea, 1958. 256 pp., 1 insert. $4.95.

Landau, L., and Smorodinsky, Ya. Lectures on nuclear theory. New York, Consultants Bureau, 1958. 82 pp. $15.00.

Landau, L. D., and Lifschitz, E. M. Course of Theoretical Physics. Vol. 3. Quantum mechanics: non-relativistic theory. Trans. by J. B. Sykes and J. S. Bell. (Addison-Wesley Series in Advanced Physics.) Reading, Addison-Wesley, 1958. 12 + 515 pp. $12.50.

Landau, L. D., and Lifšic, E. M. Èlektrodinamika splošnyh sred. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 532 pp. 11.70 rubles.

Leacock, J. A. See Boll, R. H.

Lewin, L. Dilogarithms and associated functions. Foreword by J. C. P. Miller. London, Macdonald, 1958. 16 + 353 pp. 65s.

Lifschitz, E. M. See Landau, L. D.

Lifšic, E. M. See Landau, L. D.

Lotka, A. J. Elements of mathematical biology. New York, Dover, 1958. 30 + 465 pp. $2.45.

McCarthy, P. J. See Stephan, F. F.

Magner, T. F. Manual of scientific Russian. Minneapolis, Burgess, 1958. 3 + 101 pp., 1 insert. $4.60.

Meister, M. See Kollbrunner, C. F.

Menninger, K. Zahlwort und Ziffer. Eine Kulturgeschichte der Zahl. Vol. 2. Zahlschrift und rechnen. 2d, rev. and enlarged ed. Göttingen, Vandenhoeck and Ruprecht, 1958. 11 + 314 pp.

Michel, P.-H. Les nombres figurés dans l'arithmétique Pythagoricienne. Les conférences du Palais de la Découverte. (Histoire des Sciences, Series, D.) Université de Paris, 1958. 23 pp.

Miller, J. C. P. See Lewin, L.

Mitrinović, D. S. Važnije nejednakosti. [Important inequalities.] (Matematička Biblioteka, No. 7.) Belgrade, Univerzitet u Beogradu, 1958. 64 pp.

Moulton, F. R. Differential equations. New York, Dover, 1958. 15 + 395 pp. $2.00.

Moulton, J. F. See Boole, G.

Netto, E. Lehrbuch der Combinatorik. 2d ed, enlarged and with annotations by V. Brun and Th. Skolem. New York, Chelsea, 1958. 8 + 341 pp. $4.95.

Peck, L. G., and Hazelwood, R. N. Finite queuing tables. (Publications in Operations Research, No. 2.) New York, Wiley, 1958. 16 + 3-210 pp. $8.50.

Phister, Montgomery, Jr. Logical design of digital computers. New York, Wiley, 1958. 16 + 408 pp. $10.50.

Pickert, G. Ebene Inzidenzgeometrie: Beispiele zur Axiomatik mit einer Einführung in die formale Logik. (Schriftenreihe zur Mathematik, No. 8.) Frankfurt am Main, Salle, 1958. 92 pp. 6.40 DM.

Pogorzelski, W. Równania całkowe i ich zastosowania. Vol. 2. Układy równań całkowych; równania całkowe nieliniowe; zastosowania równań całkowych w teorii równań różniczkowych. Z dodatkiem R. Sikorskiego: O twierdzeniu Schaudera. [Integral equations and their applications. Vol. 2. Systems of integral equations; non-linear integral equations; applications of integral equations to the theory of difference equations. With a supplement by R. Sikorski: On Schauder's theorem.] Warsaw, Państwowe Wydawnictwo Naukowe, 1958. 192 pp., 1 insert. 24 zł.

Proceedings of Symposia in Applied Mathematics. Vol. VIII. New York, McGraw-Hill, 1958. 153 pp. $7.50.

Proceedings of the Third Midwestern Conference on Solid Mechanics, 1957. Ann Arbor, University of Michigan Press, 1957. 6 + 250 pp. $5.50.

Progress in nuclear energy. Series 1: Physics and Mathematics. Vol. 2. New York, Pergamon, 1958. 7 + 375 pp. $14.00.

Redheffer, R. M. See Sokolnikoff, I. S.

Rheology: theory and applications. Ed. by F. R. Eirich. Vol. 2. New York, Academic Press, 1958. 591 pp. $18.00.

Richards, C. H. See Salzer, H. E.

Riley, V., and Gass, S. I. Linear programming and associated techniques. (Bibliographic Reference Series, No. 5.) Baltimore, Johns Hopkins Press, 1958. 10 + 613 pp. $6.00.

Riordan, J. An introduction to combinatorial analysis. (Wiley Publications in Mathematical Statistics.) New York, Wiley, 1958. 11 + 244 pp.

Rogers, R. A. P. See Salmon, G.

Rowe, C. H. See Salmon, G.

Ruoff, H. See Hardy, G. H.

Russian-English glossary of acoustics and ultrasonics. New York, Consultants Bureau, 1958. 3 + 171 + 23 pp. $10.00.

Salmon, G. A treatise on the analytic geometry of three dimensions. Rev. by R. A. P. Rogers. 7th ed. Vol. 1. Ed. by C. H. Rowe. New York, Chelsea, 1958. 24 + 470 pp. $4.95.

Salzer, H. E., Richards, C. H., and Arsham, I. Table for the solution of cubic equations. New York, McGraw-Hill, 1958. 15 + 160 pp. $7.50.

Sauer, R. Anfangswertprobleme bei partiellen Differentialgleichungen. 2d ed. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Vol. 62.) Berlin, Springer, 1958. 16 + 284 pp. 41.00 DM.

Scheidegger, A. E. Principles of geodynamics. Berlin, Springer, 1958. 11 + 280 pp. 49.60 DM.

Scheja, G. See Hirzebruch, F.

Sears, F. W. Mechanics, wave motion, and heat. (Principles of Physics Series.) Reading, Addison-Wesley, 1958. 13 + 664 pp. $9.50.

Segre, C. Opere. Vol. 2. Ed. by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1958. 18 + 438 pp., 1 plate. 4000 Lire.

Semendjajew, K. A. See Bronstein, I. N.

Sierpiński, W. Cardinal and ordinal numbers. (Polska Akademia Nauk, Monografie Matematyczne, Vol. 34.) Warsaw, Państwowe Wydawnictwo Naukowe, 1958. 487 pp.

Sikorski, R. See Pogorzelski, W.

Simonov, N. I. Prikladnye metody analiza u Èĭlera. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 167 pp. 6.15 rubles.

Skolem, Th. See Netto, E.

Slyusarev, G. G. O vozmožnom i nevozmožnom v optike. 2d, rev. ed. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 178 pp. 2.90 rubles.

Smorodinsky, Ya. See Landau, L.

Sokolnikoff, I. S., and Redheffer, R. M. Mathematics of physics and modern engineering. New York, McGraw-Hill, 1958. 9 + 810 pp. $9.50.

Šteĭngauz, G. See Kačmaž, S.

Stephan, F. F., and McCarthy, P. J. Sampling opinions: an analysis of survey procedure. (Wiley Publications in Statistics.) New York, Wiley, 1958. 21 + 451 pp. $12.00.

Struik, D. J. Su gaku no reki shi. [A concise history of mathematics.] Tokyo, Misuzu Shobo, 1958. 4 + 224 pp., 8 plates. 280 yen.

Sykes, J. B. See Landau, L. D.

Table of natural logarithms for arguments between five and ten to sixteen decimal places. (National Bureau of Standards. Applied Mathematics Series, No. 53.) Washington, Government Printing Office, 1958. 13 + 506 pp. $4.00.

Talypov, G. B. Približennaya teoriya cvaročnyh deformaciĭ i napryaženiĭ. Izdat. Leningradsk. Univ., 1957. 206 pp., 1 insert. 9.00 rubles.

Titchmarsh, E. C. Eigenfunction expansions associated with second-order differential equations. Vol. 2. Oxford, Clarendon, 1958. 11 + 404 pp. $11.20.

The tree of mathematics. Ed. by G. James. Pacoima, The Digest Press, 1957. 17 + 403 pp. $6.00.

Tuttle, D. F. Jr. Network synthesis. Vol. 1. New York, Wiley, 1958. 15 + 1175 pp. $23.50.

Ul'yanov, P. L. See Kačmaž, S.

Van Nostrand's scientific encyclopedia. 3d ed. Princeton, Van Nostrand, 1958. 5 + 1839 pp., 11 plates.

Vilenkin, N. Ya. See Kačmaž, S.

Wallace, A. H. Homology theory on algebraic varieties. (International Series of Monographs on Pure and Applied Mathematics, Vol. 6.) New York, Pergamon, 1958. 8 + 115 pp. $5.50.

Walz, A. Nouvelle méthode approchée de calcul des couches limites laminaire et turbulente en écoulement compressible: cas d'un gradient de pression; particularités, exemples. (Publ. Sci. Tech. Ministère de l'Air, no. 336.) Paris, 1957. 5 + 48 pp. 900 fr.

Weber, C., and Günther, W. Torsionstheorie. Braunschweig, Wieweg, 1958. 4 + 307 pp. 38.00 DM.

Weber, G. See Heber, G.

Wittenberg, A. I. Vom Denken in Begriffen: Mathematik als Experiment des reinen Denkens. With a preface by P. Bernays. (Wissenschaft und Kultur, vol. 12.) Basel, Birkhäuser, 1957. 360 pp. 25.00 DM.

Wright, E. M. See Hardy, G. H.

Zurmühl, R. Matrizen: eine Darstellung für Ingenieure. 2d ed. Berlin, Springer, 1958. 15 + 467 pp. 33.00 DM.

Zypkin, Ja. S. Theorie der Relaissysteme der automatischen Regelung. Munich, Oldenbourg, 1958. 472 pp. 52.00 DM.

ABSTRACTS OF CONTRIBUTED PAPERS

THE OCTOBER MEETING IN PRINCETON, NEW JERSEY

October 25, 1958

549-34. Smbat Abian and A. B. Brown: On the solution of simultaneous first order implicit differential equations.

The authors give an existence and uniqueness proof and an explicit method of solving a given system of n simultaneous implicit first order ordinary differential equations $f_i(x,y_1,\ldots,y_n, y'_1,\ldots,y'_n) = 0$, without solving explicitly for the derivatives. The assumed hypotheses are weaker than those ordinarily imposed. In particular, the existence of a set of values satisfying the given system is not required. A set of n functions $F_i(x,y,z) = z_i + \sum_{k=1}^{n} D_{ik} f_k(x,y,z)$ is introduced, where (D_{ik}) is an n by n matrix of constants. Functions $Y_i(x;r)$ are defined by $Y_i(x;r) = b_i + \int_a^x F_i[t,Y(t;r-1), Y'(t;r-1)]\,dt$, and the solution of the given system of differential equations is obtained as $y_i = Y_i(x) = \lim_{r\to\infty} Y_i(x;r)$. Four appraisals of the remainder error at the rth stage of approximation are given, two of which are valid regardless of errors made in the work through the (r - 1)st stage. (Received September 2, 1958.)

549-35. Abraham Charnes and C. E. Lemke: Multi-copy generalized networks.

Multi-copy generalized networks (the Kirchhoff node conditions are generalized) were introduced by Charnes and Cooper (Proc. Nat. Acad. Sci., February, 1958) in connection with extremal problems and principles in traffic flow, plastic limit analysis, financial planning, etc. (e.g. NRLQ, September, 1958, Journal of Business, September 1958 (with M. H. Miller)). Such problems in sizes typical of reality are beyond the limits of general purpose machine computation methods even for linear functionals. This paper presents a special method, exploiting the structure of such constraints, for extremizing a linear functional over a generalized network which brings them within reach. (Received August 11, 1958.)

549-36. Eckford Cohen: A trigonometric sum.

Let $k,r,n,n_1,\ldots,n_k$ denote integers, $k > 0$, $r > 0$. Let $e(n,r) = \exp(2\pi i n/r)$ and place $c_r(n_1,\ldots,n_k) = \sum e_r(n_1x_1 + \ldots + n_kx_k)$, where the summation $\{x_i\}$ is over a reduced residue system (mod k,r), that is, over x_i (mod r) such that

$(x_1,\ldots,x_k,r) = 1$. This sum is the k-dimensional analogue of Ramanujan's sum. Also place $A_r(n_1,\ldots,n_k) = \sum c_r(n_1 - a_1,\ldots,n_k - a_k)$, the summation $\{a_i\}$ being over a reduced residue system (mod k,r). Two different evaluations of the sum $A_r(n_1,\ldots,n_k)$ are deduced. Equating these two expressions yields an arithmetical relation which reduces to the Brauer-Rademacher identity in case $k = 1$. (Received July 31, 1958.)

549-37. R. S. Freeman: Self adjoint boundary conditions for the Laplacian. I. Bounded regions in the plane.

Let G be a bounded domain in E^2. Suppose ∂G is in C^1 with Lipschitz continuous first derivatives, so that G has bounded curvature. Let points in G be denoted by x and points in ∂G by θ. For $0 \leqq r \leqq r_0$, where r_0 is some fixed number, x in G can be represented in the form (r,θ) where r is a distance along the inner normal at θ. A function u in $\mathcal{L}^2(G)$ is said to have an boundary value when there exists a function $\tilde{u}$ in $\mathcal{L}^2(\partial G)$ such that $\lim_{n\to 0} \int_{\partial G} |u(r,\theta) - \tilde{u}(\theta)|^2 \, d\theta = 0$. Let $D_1(G)$ be the set of functions u in $P^2(G)$ of Morrey and Calkin (ref. Functions of several variables and absolute continuity, I, II, Duke Math. J. vol. 6 (1940) pp. 170-215) such that $-\Delta u$ is in $\mathcal{L}^2(G)$ and u and u_n its normal derivative have boundary values in the above sense. Let L be a bounded self adjoint operator on $\mathcal{L}^2(\partial G)$ and $D(T_L)$ the set of functions u in $D_1(G)$ such that $\tilde{u}_n - L\tilde{u} = 0$. Let $T_L u = -\Delta u$ for u in $D(T_L)$, then T_L is a self adjoint operator on $L^2(G)$ with compact resolvent. The method is based on a paper of Calkin (ref. J. W. Calkin, Abstract symmetric boundary conditions, Trans. Amer. Math. Soc. vol. 45 (1939) pp. 369-442). (Received September 8, 1958.)

549-38. R. S. Freeman: Self adjoint boundary conditions for the Laplacian. II. Unbounded regions in the plane.

Suppose G is unbounded but that ∂G is bounded and subject to the conditions of the preceding abstract. Suppose furthermore that $D_1(G)$, $D(T_L)$, and T_L, are defined as in the preceding abstract. Then it is shown that T_L is a self adjoint as before, this case being reduced to that of a bounded region. When ∂G is unbounded and L is multiplication by a bounded, real valued function then T_L is self adjoint under suitable additional restrictions on the behavior of ∂G near infinity. This result depends on a localization theorem due to E. T. Poulsen (Technical Report No. 25 (NR 041 157) Department of Mathematics, University of California at Berkeley). (Received September 8, 1958.)

549-39. I. S. Gál: Some properties of the Dirac delta.

The purpose of this note is to show that the following formal properties of the Dirac delta are meaningful in the theory of Schwartz distributions and simple proofs can be given by using the author's interpretation of distributions as ideal elements of the completion of a uniform structure $\mathcal{U}_S$ for the linear space $X = \mathcal{D}$. (1) $\delta(-x) = \delta(x)$; (2) $a\delta(ax) = \delta(x)$; and (3) $2a\delta(x^2 - a^2) = \delta(x - a) + \delta(x + a)$ where $a > 0$. The following lemma is essential in the proof: For each $\lambda \geq 1$ let x_λ be a function in X such that $x_\lambda \geq 0$, the smallest support of x_λ shrinks to the point 0 and $\int x_\lambda \, d\mu \to 1$ as $\lambda \to \infty$. Then $\lim(x_\lambda) = \delta$ as $\lambda \to \infty$ relative to the uniform topology associated with $\mathcal{U}_S$. The involution $I: X \to X$ defined by $(Ix)(s) = x(-s)$ is continuous with respect to $\mathcal{U}_S$ and so it can be extended to a continuous linear operator $I: \bar{X} \to \bar{X}$ where $\bar{X}$ is the space of distributions. Formula (1) means that $I\delta = \delta$. Formula (2) can be interpreted similarly. To interpret (3) prove that the map A given by $(Ax)(s) = x(s^2 - a^2)$ is continuous on a linear space $L \subset X$ whose closure contains δ. For instance L can be chosen as the space of those $x \in X$ which vanish outside of $[-a^2/4, a^2/4]$. (Received September 8, 1958.)

549-40. Leon Greenberg: A discreteness theorem for subgroups of the Lorentz group.

Let $L^+(n)$ be the group of linear transformations of degree n, which preserve the form $F(x) = \sum_{i=1}^{n-1} x_i^2 - x_n^2$, and preserve each half $x_n > 0$ and $x_n < 0$ of the isotropic cone $F(x) = 0$. $L^+(n)$ is also the conformal group of the unit ball $B^{n-1} = \{y \mid |y| < 1\}$ in Euclidean space E^{n-1}. Suppose $A \in L^+(n)$ has positive determinant and eigenvalues $\{e^{\lambda_k} \mid k = 1, 2, \ldots, n\}$. If $|e^{\lambda_k}| \neq 1$, λ_k can be chosen to be real, and if $|e^{\lambda_k}| = 1$, λ_k can be chosen purely imaginary. This determines λ_k except when $e^{\lambda_k} = -1$, when λ_k can be $\pm\pi i$. There are an even number of the eigenvalues (-1); choose half of the corresponding $\{\lambda_k\}$ to be $+\pi i$, and half to be $-\pi i$. With these choices of $\{\lambda_k\}$, define $T(A) = \sum_{i<j} \lambda_i \lambda_j$. A is called strictly parabolic, if every eigenvalue is 1 (and $A \neq$ identity). Theorem: Let G be a subgroup of $L^+(n)$. Suppose that G contains no strictly parabolic elements, that all of the elements of G do not have a common fixed point of the boundary sphere $S^{n-2} = \partial B^{n-1}$, and that they do not all map the same pair of points in S^{n-2} onto that same pair. Suppose further that there is a neighborhood of the identity in $L^+(n)$, whose intersection with G contains (besides the identity) only elements A

for which $T(A) > 0$. Then G is discrete. The argument consists mainly in Lie algebra computations. (Received June 26, 1958.)

549-41. Leon Greenberg: Hyperbolic space forms. Preliminary report.

A hyperbolic space form (h.s.f.) is a complete Riemannian manifold with constant negative sectional curvature. Theorem: Let G be the fundamental group of a h.s.f. Then: (1) For every $g \in G$, $g \neq 1$, the centralizer $C(g) = \{h \in G | hg = gh\}$ is cyclic; (2) If G is not cyclic, then G contains no nontrivial commutative subnormal subgroups; (3) G is not a direct product; it follows that: (4) If G is not cyclic, then G is semi-simple (there are no nontrivial solvable, normal subgroups); and (5) If G is not cyclic, it has trivial center. The above properties are all true for a compact h.s.f., whose fundamental group cannot be cyclic. This implies that two compact space forms cannot be homeomorphic, if their curvatures are not both positive, both negative or both zero. The proof of the theorem involves a study of the isometry group of hyperbolic space. An example is found of a 3-dimensional, compact h.s.f. whose 1-st homology group is torsion free. This is done by considering a group generated by the reflections in the faces of a tetrahedron in hyperbolic 3-space, and finding a subgroup of finite index, containing no elements of finite order. The fundamental region of the subgroup is a polyhedron with 120 congruent (nonequilateral) triangles. (Received June 26, 1958.)

549-42. E. O. A. Kreyszig: On harmonic functions of three variables.

Whittaker's integral representation of solutions of Laplace's equation $w_{xx} + w_{yy} + w_{zz} = 0$ can be used for a systematic investigation of certain classes of harmonic functions corresponding to rational and algebraic functions $f(u,\xi)$ of the complex variables ξ and $u = x + Z\xi + Z^*/\xi$, $Z = (z + iy)/2$, $Z^* = -(z - iy)/2$, in the integrand of that representation. In the case of regular functions f a one-to-one correspondence between f and the harmonic functions, regular at the origin, can be established, as was shown by S. Bergman. In the case of functions f having singularities the theory of residues leads to explicit representations of the harmonic functions. Depending on the type and location of the singularities of f the xyz-space is divided into a number of regions in each of which the harmonic function corresponding to f is given by a different expression; at the boundaries of those regions the harmonic function is discontinuous. If f has poles it corresponds to a harmonic function which is singular along certain

algebraic curves; by means of algebraic-geometrical methods these curves ca be determined directly from f. For certain classes of algebraic curves C it is possible to determine functions f such that the corresponding harmonic functio is singular precisely along C. (Received May 2, 1958.)

549-43. E. O. A. Kreyszig and John Todd: The radius of univalence of Bessel functions. I.

It is shown that for $\nu > -1$ the radius of univalence ρ_ν of the normalised Bessel function $\tilde{J}_\nu(z) = z^{1-\nu} J_\nu(z)$ is the abscissa of the first positive maximum of $\tilde{J}_\nu(x)$. The proof uses the Weierstrass product for $\tilde{J}_\nu(z)$. Bounds for ρ_ν are obtained and it is shown that ρ_ν is an increasing function of ν, tending to zero as ν tends to -1 and to infinity as ν tends to infinity. These results wer suggested by actual computations of the maps of the circles $|z| = r$, using the Datatron 205 at the California Institute of Technology. The situation when $\nu < -1$ is more complicated and the computations suggest that the critical points can lie on the imaginary axis, or be genuine complex numbers. (Receive September 8, 1958.)

549-44. D. B. Lissner: 2 × 2 matrices of trace 0.

Let R be a commutative ring with unit, F any field, and C a 2 × 2 matrix over R. The paper is concerned with the theorem: If tr(C) = 0, then C is a commutator. By purely elementary arguments (i.e., algebraic manipulations with the polynomials involved) it is shown that the theorem is true for $R = F[x]$, false for $R = F[x_1,\ldots,x_n]$, $n \geqq 3$, and true for $R = F[x,y]$ if F is algebraically closed. Finally, modifying the proof of the last theorem with some references to the theory of polynomial ideals, a proof is obtained for the case $R = F[x,y]$, where F are the real numbers. (Research was supported in part by Army Ordnance Contract DD-30-115-ORD-976 with Cornell University.) (Received September 8, 1958.)

549-45. R. T. Prosser: On the existence of a scattering operator.

Recently we established the existence of a unitary transformation function U(t) satisfying the general Schrodinger equation $dU(t)/dt = i\lambda U(t)H(t)$, with $U(0) = I$. Here H(t) is a strongly continuous one-parameter family of essentially self adjoint operators with common dense domain in a Hilbert space. Now we consider the behavior of U(t) as $t \to +\infty$. If $\|H(t)\phi\|$ is integrable in t for

each ϕ in the common domain, then the strong limit $U(+\infty) = \lim_{t \to +\infty} U(t)$ exists. If $\|H(t)\phi\|$ is merely bounded in t, then the Abel limit $W(+\infty) = \lim_{\epsilon \to 0} \epsilon \int_0^\infty e^{-\epsilon t} U(t)dt$ exists. These limits need not be unitary, but it is possible to give conditions which guarantee that they are. We conclude that in most simple scattering problems of physics these limits exist and are unitary. (Received September 9, 1958.)

549-46. J. J. Schäffer: Function spaces with translations.

The class $\mathfrak{T}$ consists of all nontrivial normed spaces $\mathfrak{F}$ of real-valued measurable functions defined on $J = [0, +\infty)$ satisfying: (a) for each bounded interval $J' \subset J$, $\int_{J'} |\phi(t)|dt \leqq \alpha(J')\|\phi\|_{\mathfrak{F}}$, for a fixed $\alpha(J')$ and all $\phi \in \mathfrak{F}$; (b) if $\phi \in \mathfrak{F}$, (i) ψ measurable, $|\psi(t)| \leqq |\phi(t)|$, implies $\psi \in \mathfrak{F}, \|\psi\|_{\mathfrak{F}} \leqq \|\phi\|_{\mathfrak{F}}$, (ii) for every $\tau \geqq 0$, the function $\theta(t) = \phi(t - \tau)$, $t \geqq \tau$, $= 0$, $0 \leqq t < \tau$, satisfies $\theta \in \mathfrak{F}, \|\theta\|_{\mathfrak{F}} = \|\phi\|_{\mathfrak{F}}$. If we also admit similar "left" translations we obtain the class $\mathfrak{T}^* \subset \mathfrak{T}$. $\mathfrak{T}$ admits a natural partial ordering but is not a lattice; $\mathfrak{T}^*$ is a lattice. The space $\mathfrak{M}$ of functions with $\|\phi\|_{\mathfrak{M}} = \sup_{t \in J} \int_t^{t+1} |\phi(u)|du < \infty$ is the largest element in $\mathfrak{T}[\mathfrak{T}^*]$ and certain spaces $\mathfrak{F}_E$ corresponding to subsets $E \subset J$ may be defined which are, in some sense, the smallest elements. Associate spaces (in the sense of Luxemburg-Zaanen) of $\mathfrak{T}^*$ - spaces are $\mathfrak{T}^*$- spaces; there are numerous results characterizing the properties of the operations of "association", "local closure" (in the space of all functions integrable on bounded sets), "completion" and the *-operation (which maps $\mathfrak{T}$ onto $\mathfrak{T}^*$) and their mutual connections. Orlicz spaces are a sub-lattice of $\mathfrak{T}^*$; their lattice structure may be studied in terms of the defining Young functions. This theory is extended to spaces of functions with values in general Banach spaces and to spaces of continuous functions. (Received September 2, 1958.)

549-47. J. L. Massera and J. J. Schäffer: Linear differential equations and functional analysis. V.

We adhere to the notation used in the preceding Parts [Bull. Amer. Math. Soc. vol. 63 (1957) pp. 236-237] and in Function spaces with translations by J. J. Schäffer [See preceding abstract]. The theorems stated in Part II may be generalized as follows: 15. Assume $A \in \mathfrak{M}(\tilde{X})$; if $\mathfrak{B}$ is a Banach space $\in \mathfrak{T}$ and if for every $f \in \mathfrak{B}(X)$ equation (2) has at least one bounded solution, the conclusion of Theorem 6 holds and moreover (2) has at least one bounded solution for each $f \in \mathfrak{L}^1(X)$. 16. Assume that equation (2) has at least one bounded solution for

each $f \in \mathcal{B}(X)$, where $\mathcal{B}$ is a Banach space $\in \mathfrak{T}$ which contains a function which does not belong to $\mathcal{L}^1$ (in particular, if $\mathcal{B}$ is an Orlicz space $\mathcal{L}_\Phi$ with $\Phi(u) = o(u)$ as $u \to 0$); assume furthermore that either $A \in \mathfrak{M}(\tilde{X})$ or equation (2) has at least one bounded solution for each $f \in \mathcal{L}^1$; then the conclusion of Theorem 4 holds and moreover equation (2) has at least one bounded solution for each $f \in \mathfrak{M}(X)$, a fortiori for each $f \in \mathcal{B}(X)$, where $\mathcal{B}$ is any Banach space $\in \mathfrak{T}$. (Received September 2, 1958.)

549-48. J. L. Massera and J. J. Schäffer: Linear differential equations and functional analysis. VI.

Notation as in the preceding Parts; $\mathcal{P}$ denotes the subspace of $\mathfrak{M}$ consisting of the periodic functions of period 1; $\mathcal{PC}$ is the subspace $\mathcal{P} \cap \mathcal{C}$ of $\mathcal{C}$. Let $A \in \mathcal{P}(\tilde{X})$, $U(t) \in \tilde{X}$ be the solution of $\dot{U} + AU = 0$, $U(0) = I$. 17. If $\|A\|_{\mathfrak{M}} < \log 4$ there is a constant $B \in \tilde{X}$ and a $P \in \mathcal{PC}$ such that $U(t) = P(t)e^{-tB}$ (Floquet representation; for $\|A\|_{\mathfrak{M}} > \pi$ such a representation is not always possible). 18. Equation (1) has no nontrivial solution $\in \mathcal{PC}$ if and only if $I - U(1)$ is one-to-one; equation (2) has at least one periodic solution for every $f \in \mathcal{P}$ [$\in \mathcal{PC}$] if and only if $I - U(1)$ is onto (both conditions coincide for finite-dimensional X). 19. If for a certain $f \in \mathcal{P}$ equation (2) has a bounded solution $x(t)$, it has a solution $\in \mathcal{PC}$ if one of the following supplementary assumptions is satisfied: (a) the closed convex hull of the set $\{x(n)\}$ is weakly compact; (b) the closure of X_0 is reflexive; (c) $x(t)$ is positively recurrent, in particular, asymptotically almost periodic; (d) the range of $I - U(1)$ is closed. 20. If X_0 is closed and has a closed complement and if (2) has a bounded solution for every $f \in \mathcal{B}$, a Banach space $\in \mathfrak{T}$ which contains a function $\notin \mathcal{L}^1$, then it has a unique solution $\in \mathcal{PC}$ for every $f \in \mathcal{P}$. (Received September 2, 1958.)

549-49. Martin Schechter: General boundary value problems for elliptic partial differential equations. I.

Let E_t^n denote the halfspace $x_n > 0$ in E^n and let A be a homogeneous linear partial differential operator of order 2r with constant coefficients. A is elliptic if its characteristic polynomial $P(\xi,\eta) \neq 0$ for real $(\xi,\eta) \neq 0$ where $\xi = (\xi_1,\xi_2,\ldots,\xi_{n-1})$ corresponds to $(x_1,x_2,\ldots,x_{n-1})$ and η to x_n. A is strongly elliptic if there is a complex r such that $\operatorname{Re} P(\xi,\eta) \neq 0$ for real $(\xi,\eta) \neq 0$. It is properly elliptic if for every real $\xi \neq 0$, r of the complex roots of $P(\xi,z) = 0$ lie above the real axis and r below. All strongly elliptic operators and all

elliptic operators in higher than two dimensions are properly elliptic. Let $\{B_t\}$ be a set of operators of orders $< 2r$ with characteristic polynomials $Q_t(\xi,\tau)$. For every $\xi \neq 0$ let $\lambda_1(\xi), \lambda_2(\xi), \ldots, \lambda_r(\xi)$ be the roots of $P(\xi,z) = 0$ with positive imaginary parts. The operators B_t cover A if for every $\xi \neq 0$ there are r polynomials $Q_t(\xi,z)$ which are linearly independent modulo $(z-\lambda_1)(z-\lambda_2)\ldots(z-\lambda_r)$. (Received August 25, 1958.)

549-50. Martin Schechter: General boundary value problems for elliptic partial differential equations. II.

Let $0 < p_1 < p_2 < \ldots < p_r \leq 2r$ be a sequence of integers and consider the operators $M_t = D_n^{t-1}$, $t = p_1, p_2, \ldots, p_r$, $N_t = \sum_{|\mu|+k=2r} a_\mu D^\mu D_n^{k-t}$, $t \neq p_j$, where $D_n = \partial/\partial x_n$, D^μ is any derivative of order $|\mu|$ not involving x_n and a_μ is the coefficient of $D^\mu D_n^k$ in A (cf. preceding abstract). Problem (A,f,u_0,M_t) is to find a function $u \in C_{2r}(E_t^n)$ all of whose derivatives of orders up to 2r are in $L_2(E_t^n)$ and such that $Au = f$ in E_t^n and $M_t(u - u_0) = 0$ on $x_n = 0$ for all M_t. It is proved that if A is properly elliptic and the M_t cover it, then both problems (A,f,u_0,M_t) and (A,f,u_0,N_t) have unique classical solutions provided f and u_0 are sufficiently smooth. (Received August 25, 1958.)

549-51. Maurice Sion: On integration of 1-forms.

In Euclidean N-space, let C be the image of the unit interval by a continuous function ϕ. Let m be a positive integer, $\eta > 0$, and for $k = (k_0,\ldots,k_N)$, the k_i non-negative integers, let $\sigma k = \sum k_i$. For each such k with $1 \leqq \sigma k \leqq m$, $x \in C$, $y \in C$, let $\omega_k(y) = \sum_{\sigma j=0}^{m-\sigma k} \omega_{k+j}(x)((y_0 - x_0)^{j_0} \ldots (y_N - x_N)^{j_N})/(j_0!\ldots j_N!) + R_k(x,y)$ where $|R_k(x,y)| < K|y - x|^{m+\eta-\sigma k}$ for some K. Thus, ω is a 1-form possessing 'derivatives' up to order m on C with Taylor expansions having 'Lipschitzian' remainders. Let $V_\alpha(a,b) = \sup \sum |\phi(t_{i-1}) - \phi(t_i)|^\alpha$ over all finite subdivisions of [a,b]. Using sums of polynomials of degree m and special subdivisions, it is shown that $\int_a^b \omega d\phi$ exists if $V_\alpha(a,b) < \infty$ for some $\alpha < m + \eta$. If in addition $V_\alpha(a,b) < L|\phi(b) - \phi(a)|^\alpha$ locally, then this integral is an 'anti-derivative' of ω on C having a Taylor expansion with remainder of order α. (Received August 20, 1958.)

549-52. A. H. Stone: Metrisability of unions of spaces.

Let a topological space S be a union of metrisable subspaces S_α; conditions are given which imply that S is itself metrisable. Typical results: If S is collectionwise normal and locally countably compact, the subsets S_α are closed, and the system $\{S_\alpha\}$ is σ-locally finite, then S is metrisable; this includes an analogue for nonseparable spaces of a theorem of Smirnov (Fund. Math. vol. 43 (1956)). If S is regular, each S_α is open and has a compact frontier, and the family $\{S_\alpha\}$ is countable, then S is metrisable. If S is regular, the sets S_α open and (locally) separable, and $\{S_\alpha\}$ is point-countable, then S is metrisable. (Received August 8, 1958.)

549-53. J. L. Walsh: Note on least-square approximation to an analytic function by polynomials, as measured by a surface integral.

Let D be the interior of an analytic Jordan curve C, and let $L_2(k,\alpha)$ be the class of functions f(z) in H_2 in D such that $f^{(k)}(z)$ exists on C and satisfies on C a mean square Lipschitz condition of order α $(0 < \alpha < 1)$ or Zygmund condition $(\alpha = 1)$; for $k < 0$ the (-k)th integral of f(z) shall satisfy such a condition. A necessary and sufficient condition that $f(z) \in L_2(k,\alpha)$, $k + \alpha + 1/2 > 0$, is that f(z) can be approximated in mean square on D by polynomials in z of respective degrees n with error $O(n^{-k-\alpha-1/2})$. (Received September 8, 1958).

549-54. H. C. Wang: Compact transformation groups of n-sphere with an (n - 1)-dimensional orbit.

Let G be a compact and connected group acting continuously on an n-sphere S^n with an (n - 1)-dimensional orbit X. Montgomery-Zippin proved that with two exceptions X_1, X_2, all the orbits under G are homeomorphic with X. They have analyzed the situation when X_1, X_2 are points. In this paper, the following are proved: (I) The action of G on S^n is linear. Except for five particular cases, the special orbits X_1, X_2 are either both points or both spheres S^p, S^q with $p + q = n - 1$. (II) In the particular cases, the transformation group $\{G, S^n\}$ is obtained from either (a) the linear adjoint representation of simple compact groups of rank 2, or (b) the 14-dimensional irreducible representation of the compact symplectic group C_3 of rank 3, or (c) the 26-dimensional irreducible representation of the exceptional simple group F_4. (Received July 28, 1958.)

549-55. Harold Widom: Inversion of Toeplitz matrices. II.

If $\phi(\theta) \sim \sum c_k e^{ik\theta}$ is in $L_\infty(0,2\pi)$, the semi-infinite Toeplitz matrix $T_\phi = (c_{j-k})_{0 \leq j,k < \infty}$ is a bounded operator on the space of square-summable sequences $\{x_0, x_1, \ldots\}$. Conditions are obtained for T_ϕ to have a bounded inverse. THEOREM I. For real ϕ, T_ϕ is invertible if and only if ess sup $\phi < 0$ or ess inf $\phi > 0$. Let L_∞^+ be the set of those L_∞ functions f for which $\int_0^{2\pi} f(\theta) e^{-ik\theta} d\theta = 0$ for $k < 0$; L_∞^- is the set of those L_∞ functions f for which $\int_0^{2\pi} f(\theta) e^{-ik\theta} d\theta = 0$ for $k < 0$. THEOREM II. If $\phi \in L_\infty^+$ (resp. L_∞^-) then T_ϕ is invertible if and only if $1/\phi \in L_\infty^+$ (resp. L_∞^-). Call a continuous periodic $f(\theta)$ nice if either (a) f has an absolutely convergent Fourier series; or (b) denoting the modulus of continuity of f by $\omega(\delta)$, $\omega(\delta)/\delta$ is integrable near 0. THEOREM III. Assume there may be defined an arg $\phi(\theta)$ which is periodic and continuous except for finitely many jumps at $\theta_1, \ldots, \theta_k \pmod{2\pi}$, $\arg \phi(\theta_j +) - \arg \phi(\theta_j -) = 2\pi\alpha_j$. Write $\alpha_j = \beta_j + \gamma_j$ where β_j is an integer and $-1/2 < \gamma_j \leq 1/2$. Assume the continuous function $\arg \phi(\theta) + \sum_j \alpha_j \{\theta - \theta_j - 2\pi[(\theta - \theta_j)/2\pi]\}$ is nice. Then T_ϕ is invertible if and only if (i) $1/\phi \in L_\infty$, (ii) each $\gamma_j < 1/2$, and (iii) $\sum_j \beta_j = 0$. COROLLARY. If $\phi(\theta)$ is nice then T_ϕ is invertible if and only if $\phi(\theta) \neq 0$ and $\Delta_{-\pi \leq \theta \leq \pi} \arg \phi(\theta) = 0$. (Received August 13, 1958.)

549-56. Jacob Wolfowitz: Strong converse of the coding theorem for discrete finite-memory channels.

Let C be the capacity, as defined independently by Feinstein (in a paper to appear) and the present author (Illinois Journal of Mathematics, September, 1958), of a discrete finite-memory channel. Let n be the number of letters in a word, $0 < \lambda \leq 1$, and $N(n,\lambda)$ the maximum length of an error correcting code such that the probability that any word will be received incorrectly is $\leq \lambda$. Theorem: We have $\limsup (1/n) \log N(n,\lambda) = C$. This extends to more general channels the strong converse of the coding theorem obtained independently by Feinstein (loc. cit.) and the author (loc. cit.) for the channel studied by Khintchine in Uspehi Mat. Nauk., September, 1956, and the author in Illinois Journal of Mathematics, December, 1957. (Received September 5, 1958.)

549-57. J. N. Younglove: Concerning dense metric subspaces of certain nonmetric spaces.

Let Σ denote a space satisfying R. L. Moore's Axioms 0 and 1. Σ contains a complete metric subspace which is dense in itself. This is proved by

showing that if α is an Axiom 1 sequence, i.e., a sequence of collections of regions with properties as stated in Axiom 1, then the set M_α, consisting of all points where α satisfies a stronger form of Axiom 1, is an inner limiting set dense in the set of all points. Suppose M is a point set and it is true that if G is a collection of domains covering S then there is a collection H of domains covering S such that (1) each domain of H is a subset of some domain of G and (2) if P is a point of M and $h_1, h_2, h_3, \ldots$, is an infinite sequence of distinct domains of H and for each n, A_n and B_n are points of h_n and the sequence $A_1, A_2, A_3, \ldots$ converges to P, then the sequence $B_1, B_2, B_3, \ldots$ converges to P. The set M is said to possess Property Q. It is shown that if M is a point set possessing Property Q, then there is an Axiom 1 sequence α such that M_α contains M. From this it follows that every paracompact space satisfying Axioms 0 and 1 is metric. A sufficient condition is given on a point set M in order that there exist an Axiom 1 sequence α such that M_α is M. (Received August 15, 1958.)

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November 21-22, 1958

550-1. R. L. Vaught: Universal relational systems for elementary classes and types.

The generalized continuum hypothesis is assumed throughout. Let K be a class of similar (finitary) relational systems $\mathcal{A}=(A,R_0,\ldots,R_n,\ldots)$ $(n<\gamma)$, where $\gamma\leqq\omega$. B. Jónsson (Math. Scand. vol. 4 (1956) pp. 193-208 -- whose notation is used) showed that, if K fulfills certain (purely mathematical) conditions I-VI$_1$, $\alpha>0$, and $\gamma<\omega$, then there exists an $(\aleph_\alpha,K)$ universal system $\mathcal{A}$--i.e., a system $\mathcal{A}$, of power $\aleph_\alpha$, in which every member of K of power $\leqq\aleph_\alpha$ is isomorphically imbeddable. His proof extends almost immediately to the case $\gamma=\omega$. In Tarski's notation (Proceedings of the International Congress of Mathematicians, 1950, vol. 1, pp. 705-720), K is an EC_δ class if it consists of all models of a set T of first order (with $=$) sentences; K is an ET class if there is such a T which is, moreover, complete. By applying Jónsson's result together with some metamathematical arguments, the following have been obtained: Theorem 1. Every ET class K, having an infinite member, has an $(\aleph_\alpha,K)$ universal system, for each $\alpha>0$. Theorem 2. If $\alpha>0$ and $K\in EC_\delta$, then, in order that an $(\aleph_\alpha,K)$ universal system exist, a sufficient and (clearly) necessary condition is that any two finite or denumerable members of K be isomorphically imbeddable in a member of K. (Received September 15, 1958.)

550-2. Simon Kochen: Filtration systems. II

(For terminology see Abstract 549-24 NOTICES Amer. Math. Soc., October, 1958.) Various rings of continuous functions from a topological space into the reals and into the complex numbers are proved to be filtration systems. For example, the ring of all continuous functions from a completely regular space into the reals forms a filtration system. These results lead to a class of continuity transforms in the sense of A. Robinson (See A. Robinson, Complete theories). A free filtration system of rank α over a relational system A is a filtration system F whose quotient systems F/D under prime dual ideals D in B(F) yield all arithmetic extensions of A of cardinality $\leqq\alpha$. The existence of free filtration systems for all A and α is proved. Prime ideals of free filtration

rings over algebraically closed fields and real closed fields are constructed. These constructions lead to a simple proof of a representation theorem of A. H. Lightstone and A. Robinson for Herbrand functions over algebraically closed fields. (See J. Symbolic Logic, vol. 22.) In the case of the direct product mentioned in Filtration Systems I (Abstract 549-24) work has also been done by J. Łos, D. Scott, and A. Daigneault. (Received September 10, 1958.)

550-3. D. V. V. Wend: Some nonoscillation theorems on the real axis for complex differential equations.

Green's function methods are used to obtain nonoscillation and distribution of zeros theorems for (*) $(p(x)y')' + f(x)y = 0$ and (**) $y''' + f(x)y = 0$ on intervals $I = E(x|x_0 \leqq x < \infty)$, where $f(x)$ is complex-valued and continuous on I. If $p(x) > 0$ is of class C' on I, then (*) is nonoscillatory on I if either (1) $\int_{x_0}^{\infty}|f(x)|dx\int_{x_0}^{\infty}(1/p(x))dx < \infty$, (2) $\int_{x_0}^{\infty}|f(x)|\int_{x_0}^{x}(1/p(t))dt)dx < \infty$ or (3) $\int_{x_0}^{\infty}|f(x)|\int_{x}^{\infty}(1/p(t))dt)dx < \infty$. If $p(x)$, of class C', is complex-valued and possesses the property that for any numbers $a < b < c$ on I, both $\left|\int_a^b(1/p(x))dx\right| < \left|\int_a^c(1/p(x))dx\right|$ and $\left|\int_b^c(1/p(x))dx\right| < \left|\int_a^c(1/p(x))dx\right|$, then (*) is nonoscillatory if any one of conditions (1), (2) or (3) above is satisfied with p replaced by $|p|$. As examples of the type of distribution of zeros theorems obtained, if $\int_{x_0}^{\infty}|f(x)|dx = K < \infty$ and a_n is the n-th consecutive zero of a solution of (*), $a_1 \geqq x_0 > 0$, then if $p \equiv x$, $a_n \geqq x_0\exp(4(n-1)/K)$, while if a_n is the n-th consecutive zero of a solution of (**), then $a_n > ([n - 1 - (1+(-1)^n/2)]/2K)^{1/2}$, $n >$ (Received October 1, 1958.)

550-4. E. O. Thorp: Projections onto the subspace of compact operators

Problem: Given a pair of Banach spaces X and Y, consider the Banach space of bounded operators on X into Y. Does there exist a bounded projection onto the closed subspace of compact operators? The main result is that there are no bounded projections onto the subspace of compact operators in the following cases: 1. $X = \ell^p(S)$, $1 \leqq p < \infty$ and $Y = \ell^q(T)$, $1 \leqq p \leqq q \leqq \infty$ or $c_0(T)$ or $c(T)$. 2. $X = c_0(T)$, $Y = \ell^\infty(T)$, $c_0(T)$, $c(T)$. 3. $X = c(S)$, $Y = \ell^\infty(T)$ (Received October 2, 1958.)

550-5. Robert Steinberg: Variations on a theme of Chevalley.

The structural results obtained by Chevalley (Tôhoku Math. J. vol. 7 (1955) pp. 14-66) for the analogues over arbitrary fields of the simple complex Lie groups are extended to analogues of certain simple real Lie groups of types

A_ℓ, D_ℓ, and E_6, as well as to some groups constructed from D_4 via the principle of triality and field extensions of degree 3. Simplicity is proved under mild restrictions on the number of elements of the base field. Among these simple groups are new finite ones of orders $u^{-1}q^{36}(q^2 - 1)(q^5 + 1)(q^6 - 1)(q^8 - 1)$ $\cdot(q^9 + 1)(q^{12} - 1)$ and $q^{12}(q^2 - 1)(q^6 - 1)(q^8 + q^4 + 1)$, where q is an arbitrary prime power not less than five and u is the greatest common divisor of 3 and $q + 1$. (Received October 2, 1958.)

550-6. Alfred Tarski: Some model-theoretical results concerning weak second-order logic.

Let F be the ordinary first-order logic with identity, with first-order variables x,y,... assumed to range over elements of a set U. Let WS be the weak second-order logic obtained by enriching the vocabulary of F by means of: (i) second-order variables X, Y,... assumed to range over all finite sequences of elements of U; (ii) two logical constants, I and $\frown$, where Ix denotes the sequence containing x as the only term, and $X \frown Y$ denotes the juxtaposition of the sequences X and Y. It is known when an F-sentence (possibly containing nonlogical symbols) holds in a mathematical structure $\mathfrak{A}$, when two structures $\mathfrak{A}$ and $\mathfrak{B}$ are F-equivalent (arithmetically equivalent), and when a structure $\mathfrak{A}$ is an F-extension (arithmetical extension) of a substructure $\mathfrak{B}$; cf. Tarski-Vaught, Compositio Mathematica vol. 13, p. 81. The corresponding WS-notions are defined analogously. With the exception of 1.11 and 2.2, all the results of Tarski-Vaught extend to WS-notions. Various applications to special structures given there can actually be improved; e.g., two free groups are WS-equivalent iff either both have the same finite number of free generators or each has infinitely many free generators. (Received October 3, 1958.)

550-7. T. E. Frayne, D. S. Scott and Alfred Tarski: Reduced products.

For each $i \in I$, let $\mathfrak{A}_i = \langle A_i, R_i \rangle$ be a relational system. For simplicity each R_i will be assumed to be a ternary relation on A_i, i.e., $R_i \subseteq A_i^3$. Let J be a (proper) ideal in the field of all subsets of I. In the Cartesian product P of the sets A_i, J induces an equivalence relation, $\equiv_J$, defined by the condition: $f \equiv_J g$ iff $\{i | f(i) \neq g(i)\} \in J$. Let R_J be the relation such that $R_J(f,g,h)$ iff f,g, $h \in P$ and $\{i | \sim R_i(f(i), g(i), h(i))\} \in J$. The equivalence $\equiv_J$ is a congruence relation in the relational system $\mathfrak{P} = \langle P, R_J \rangle$ and induces the coset relational system $\mathfrak{P}_J\langle \mathfrak{A}_i | i \in I \rangle = \mathfrak{P}/\equiv_J$. The system $\mathfrak{P}_J\langle \mathfrak{A}_i | i \in I \rangle$ is called the reduced (direct) product of the $\mathfrak{A}_i$'s under J. If $\mathfrak{A}_i = \mathfrak{A}$ for all $i \in I$, $\mathfrak{P}_J\langle \mathfrak{A}_i | i \in I \rangle$

will be called a reduced power of $\mathcal{A}$. In case J is a prime ideal, $\mathcal{P}_J\langle\mathcal{A}_i|i\in I\rangle$ will be called an ultraproduct. Łoś in Mathematical interpretation of formal systems, p. 103 ff, defines an operation on algebraic systems which is essentially equivalent to constructing ultraproducts. Chang and Morel in On closure under direct product, (J. S. L., vol. 23) have discussed the particular reduced product in which the ideal J consists of all finite subsets of I. (Received October 6, 1958.)

550-8. T. E. Frayne, A. E. Morel and D. S. Scott: Set-theoretical properties of reduced products.

For notation, see this issue Abstract 550-7. Reduced products satisfy a general associative law; hence a reduced product of reduced products is isomorphic to a reduced product of the original factors. A similar conclusion holds for ultraproducts. If an ideal J in the field of all subsets of I is included in an ideal J', then $\mathcal{P}_{J'}\langle\mathcal{A}_i|i\in I\rangle$ is a homomorphic image of $\mathcal{P}_J\langle\mathcal{A}_i|i\in I\rangle$. A reduced product of subsystems can be embedded in the corresponding reduced product of the systems. Further, any system can be embedded in any reduced power of the system as well as in certain ultraproducts of its finite subsystems; if a reduced product is countable, then it is isomorphic to a finite direct product of certain of its factors. An ultraproduct of infinite factors is at least the power of the continuum unless it is isomorphic to one of the factors, in which case the ideal must be countably complete. Ultrapowers of an infinite system may be made arbitrarily large. (Received October 6, 1958.)

550-9. A. C. Morel, D. S. Scott and Alfred Tarski: Reduced products and the compactness theorem.

For notation, see this issue, Abstract 550-7. I. Let $\langle K_n|n\in\omega\rangle$ be an infinite decreasing sequence of nonempty elementary classes; by the compactness theorem, the intersection K of all K_n's is nonempty. Given $\mathcal{A}_n\in K_n$ for each $n\in\omega$, a system $\mathcal{A}\in K$ is constructed in mathematical terms: Let $\mathcal{A} = \mathcal{P}_J\langle\mathcal{A}_n|n\in\omega\rangle$, where J is any prime ideal containing the ideal F of finite subsets of ω. If all K_n's are Horn classes, let J be any ideal containing F. Generalization: Let $\langle K_i|i\in I\rangle$ be a system of elementary classes closed under finite intersection, with $\mathcal{A}_i\in K_i$ for $i\in I$. Put $\mathcal{A} = \mathcal{P}_J\langle\mathcal{A}_i|i\in I\rangle$, where J is any prime ideal including all sets $J_i = \{j|j\in I \text{ and } \mathcal{A}_j\notin K_i\}$; if all K_i's are Horn classes, let J be any ideal containing the J_i's. II. Analogously, one obtains a

mathematical realization of Henkin's theorem (Trans. A. M. S., 74, p. 414). III. Let K be an elementary (resp. Horn) class, J a prime ideal (resp. ideal) in the field of subsets of I. If $\{i | i \in I \text{ and } \mathfrak{A}_i \notin K\} \in J$, then $\mathfrak{P}_J \langle \mathfrak{A}_i | i \in I \rangle \in K$. The converse holds (but not for Horn classes and arbitrary ideals). Consequently every elementary class is closed under ultraproducts (Łoś, Mathematical interpretation of formal systems, p. 105), every Horn class closed under reduced products (C. C. Chang). (Received October 6, 1958.)

550-10. T. E. Frayne and D. S. Scott: Model-theoretical properties of reduced products.

For notations, see this issue Abstract 550-7, as well as Tarski, Indag. Math. vol. 16, p. 572 ff. and Tarski-Vaught, Compositio Mathematica vol. 13, p. 85 ff. The following results hold: I. Every ultrapower of a system $\mathfrak{A}$ is isomorphic to an elementary extension of $\mathfrak{A}$. II. If $\mathfrak{A}$ is elementarily equivalent to $\mathfrak{B}$, then some ultra-power of $\mathfrak{A}$ is isomorphic to an elementary extension of $\mathfrak{B}$. III. A class K is an elementary class in the wider sense iff K is closed under isomorphism, elementary subsystems and ultraproducts. IV. A class K is quasi-elementary iff K is closed under isomorphism, elementary extension, ultraproducts, and there exists a cardinal α such that each system in K has an elementary subsystem in K of power at most α. The proofs of Theorems III and IV make use of Theorem III of Abstract 550-9, this issue. (Received October 6, 1958.)

550-11. C. E. Burgess: Some conditions under which a homogeneous continuum is a simple closed curve.

This paper extends some results that appeared in a recent paper [Continua and various types of homogeneity, Trans. Amer. Math. Soc. vol. 88 (1958) pp. 366-374]. The following theorems are proved. (1) A nondegenerate continuous curve is a simple closed curve provided it is nearly homogeneous and is not a triod. (2) A nondegenerate compact metric continuum is a simple closed curve provided it is 2-homogeneous and is not a triod. (3) A nondegenerate bounded continuum M is a simple closed curve provided it is homogeneously embedded in a plane E; that is, for any two points x and y of M, there is a homeomorphism of E onto itself that carries M onto itself and x into y. This last result depends strongly upon a classification of homogeneous decomposable bounded plane continua given by F. B. Jones [On a certain type of homogeneous plane continuum, Proc. Amer. Math. Soc. vol. 6 (1955) pp. 735-740]. (Received October 6, 1958.)

550-12. R. A. Beaumont and R. J. Wisner: The nucleus of a torsion-free abelian group.

Let R be the additive group of rational numbers, G be a torsion-free abelian group, and let $x \in G$. Then $R_x = [r \in R | rx \in G]$ is an integer containing subgroup of R. The nucleus of G is the group $D = \bigcap_{x \in G} R_x$. D is an integer containing subgroup of R. We use this concept to prove the following theorem: Let G be a torsion-free abelian group of rank 2. Then $\mathcal{R}$ is a noncommutative ring with G as its additive group if and only if multiplication in G is defined by $X \cdot Y = f(X)Y$ for $X, Y \in G$, where f is a nontrivial homomorphism of G into the nucleus D of G. (Received October 6, 1958.)

550-13. H. S. Bear: Some boundary properties of maximal algebras.

Let A be an essential maximal closed subalgebra of C(X) for compact X, and let Σ be the maximal ideal space of A. A maximum set (zero set) is a set of the form $\{x \in \Sigma: f(x) = \|f\|\}$ ($\{x \in \Sigma: f(x) = 0\}$) for some nonconstant $f \in A$. A representing measure is a positive measure on X such that $\int f dm = f(x)$ for some $x \in \Sigma \sim X$ and all $f \in A$. Let $A|E$ be the set of all restrictions to E of functions in A. Theorem 1. If E is a maximum set, then $A|E = C(E)$. Theorem 2. If E is a closed subset of X and $A|E = C(E)$, then E is contained in some zero set. Theorem 3. If E is a maximum set, then $m(E) = 0$ for every representing measure m. Theorem 4. If the real parts of functions in $A|X$ are uniformly dense in all continuous real functions on X and E is a zero set which is contained in X, then $m(E) = 0$ for all representing measures m. (Received October 6, 1958.)

550-14. H. J. Keisler: The theory of models with generalized atomic formulas. I.

A new method in the theory of models of a lower predicate logic with identity is presented; it consists in generalizing the notion of atomic formulas. A set F of formulas is generalized atomic iff it is closed under substitution of individuals and logical equivalence, and contains the formula $x = y$. Generalizations of model-theoretic results of Birkhoff, Henkin, and Tarski are obtained. E.g., define $\mathfrak{A} = \langle A, R \rangle$ as an F-subsystem of $\mathfrak{B} = \langle B, S \rangle$ iff $A \subseteq B$ and whenever $\langle a_1, \ldots, a_n \rangle \in A^{(n)}$, $f \in F$, and $f(a_1, \ldots, a_n)$ holds in $\mathfrak{B}$, $f(a_1, \ldots, a_n)$ holds in $\mathfrak{A}$. It is proved that if $K \in AC_\Delta$, the class of F-subsystems of models of K is the class characterized by all sentences universal in F and holding throughout K. Analo-

gous theorems are proved for sentences universal positive in F, existential in F, etc. Using these results, new proofs are obtained for the theorem of Los'-Suszko and Chang on unions of chains, and of Lyndon on homomorphisms, the latter proof avoiding Gentzen-type arguments. These, too, are proved for generalized atomic formula sets. These methods yield the theorem: $\mathfrak{R}$ and $\mathfrak{F}$ are arithmetically equivalent iff an arithmetical extension of $\mathfrak{R}$ is isomorphic to one of $\mathfrak{F}$. (Received October 6, 1958.)

550-15. D. H. Tucker: Absolutely continuous solutions of a certain nonlinear hyperbolic partial differential equation.

Definitions concerning nondecreasing and bounded variation for functions of two variables, similar to those used by Hardy, Krause, Fréchet and others, are used in the development of a derivative operator for functions of two variables. These are used to establish an existence theorem concerning the equation $u_{xy} = f(x,y,u,u_x,u_y)$, where f is continuous and satisfies a Lipschitz condition in its last two variables, and boundary conditions which satisfy a Lipschitz condition. The method used is an analogue of the Euler-Cauchy polygon method, similar to that used by Diaz (NAVORD report 4451, January 16, 1957). The derivative operator developed is used to replace the cross derivative in the partial differential equation. (Received October 6, 1958.)

550-16. Anil Nerode: Remarks on isolic arithmetic. II.

First, the results of Abstract 544-29 are strengthened to show that: for isols X, Y, $(X!)^Y Y!$ divides $(XY)!$; for F a polynomial with rational integral coefficients and $n \in N = \{0,1,2,\ldots\}$, $F(x_1,\ldots,x_k) \equiv 0 \bmod n$ for all $x_1,\ldots,x_k \in N$ implies $F(X_1,\ldots,X_k) \equiv 0 \bmod n$ for all isols $X_1,\ldots,X_k$. Next, various operations of infinite addition and multiplication are defined for recursive sequences of isolic functions which are recursive combinatory (rcf) in the sense of Myhill. (They include compositions of $X + Y$, XY, X^Y, $X!$, $\binom{X}{n}$, $A^{X-n}B^n\binom{X}{n}$.) Metatheorems extend infinite identities from N to isols: for example, for isols A, B, X, Y, $(A + B)^X = \sum_{n \in N} A^{X-n}B^n\binom{X}{n}$ and $((X + 1)(Y + 1))! = (X + 1)!^{Y+1}(Y + 1)!^{\prod_{n \in N}(X+(X+1)(Y-n))!/X!((X+1)(Y-n))!}$. Other metatheorems reduce the decision problem for certain classes of elementary sentences about isols to problems in the arithmetic of N. For a simple example, let P, Q, R, S: $N^k \to N$ be rcf. If $P = Q \Rightarrow R = S$ is satisfied for all but a finite subset T of N^k, then $P = Q \Rightarrow R = S$ is satisfied for all k-tuples of isols not in T. The situation is

similar for all universal Horn sentences concerning equations between rcf. Myhill has shown this to be "best possible" for universal sentences. (Received October 6, 1958.)

550-17. W. T. Kyner and Milton Lees: The structure of electrical networks.

The purpose of this paper is to present a criterion for the solvability of the algebraic equations which are associated with a network problem. The following theorem is proved: If Z is a nonsingular b × b matrix and e is an arbitrary vector in a complex b-dimensional vector space E_b, then for every direct sum decomposition of E_b into orthogonal subspaces E_m and E_r, the system of equations $Zi + w = e$ has a unique solution with i in E_m and w in E_r if and only if one of the following statements obtains: (a) if D is a b × m matrix of rank $m \leqq b$, then (D^*ZD) is nonsingular, (b) $x \neq 0$ implies that the quadratic form $(x,Zx) \neq 0$, (c) there exists a real number θ such that $e^{j\theta}Z$ ($j^2 = -1$) is positive definite, i.e. for all $x \neq 0$, real part of $(x,e^{j\theta}Zx) > 0$, (d) $e = 0$ implies $i = 0$. (Received October 6, 1958.)

550-18. R. L. McKinney: Positive bases for linear spaces.

Positively spanning sets and positive bases (Davis, Theory of positive linear dependence, Amer. J. Math., vol. 76, 1954) are studied for an arbitrary real linear space L. One result shows that if A positively spans an infinite-dimensional L, $B \subset A$, and card B < card A then $A \sim B$ is linearly dependent. Necessary and sufficient conditions involving convex cones and semispaces are given to insure that a subset positively spans L. The theorem of Davis characterizing positive bases is generalized. Members of a large class of positive bases are shown to provide essentially unique representations of elements of L. (Received October 6, 1958.)

550-19. J. H. Walter: Structure of cleft rings, I.

Let R be a cleft semiprimary ring; that is, as an additive group $R = S \oplus N$ where S is a semisimple ring with minimum condition and N is its Jacobson radical. Then R may be regarded as an algebra over a field K as it is a direct sum of such algebras. Let $\oplus_{i=1}^{k} S_i$ be the decomposition of S into simple ideals S_i. Let F_i, $i = 1,2,\ldots,k$, be the irreducible left R-modules such that $S_iF_i \neq 0$. Let K_i be the corresponding endomorphism sfields. Let U_i be the indecomposable

left ideals of R such that U_i/NU_i is isomorphic to F_i. A structure $\sum(R,S)$ for R is a set of bilinear functions γ_{ji}^{ξ}, i,j,ξ = 1,2,...,k such that γ_{ji}^{ξ}: $\mathrm{Hom}_S(U_\xi, F_j) \times \mathrm{Hom}_S(F_i, U_\xi) \longrightarrow H_{ji}$ where H_{ji} is the double (K_j,K_i)-module of double (S,S)-module homomorphisms $\mathrm{Hom}_{(S,S)}(R, \mathrm{Hom}_K(F_i,F_j))$. Let R' = S' ⊕ N' be a second cleft ring with corresponding modules, ideals, etc. denoted by F_i', U_i', etc. Suppose that I_0: S ⟶ S' is an isomorphism of rings. Then there is induced isomorphisms I_i: $K_i \longrightarrow K_i'$, i = 1,2,...,k, and I_i-isomorphisms ϕ: $\mathrm{Hom}_S(F_i,U_\xi) \longrightarrow \mathrm{Hom}_{S'}(F_i',U_\xi')$ and ϕ^*: $\mathrm{Hom}_S(U_\xi,F_i) \longrightarrow \mathrm{Hom}_{S'}(U_\xi',F_i')$, i, = 1,2,...,k. The principal theorem of this paper states that a necessary and sufficient condition for I_0 to be extended to an isomorphism I: R ⟶ R' is that there exists (I_j,I_i)-isomorphisms θ:$H_{ji} \longrightarrow H_{ji}'$ such that $\theta\gamma_{ji}^{\xi}[F,f] = \gamma_{ji}^{\xi'}[\phi^*f^*,\phi f]$. Other results are given. This extends and completes results of Bull. Amer. Math. Soc. Abstract 62-6-640. (Received October 7, 1958.)

550-20. C. R. Karp: Split semantic models.

Notation refers to Abstract 542-57. α and β are assumed cardinal numbers such that α is regular and $\alpha \geq \beta \geq \omega$. Models for languages $F_{\alpha\beta}$ are introduced where the domain of individuals is split into an infinite sequence of domains. If $\beta = \omega$ validity in split semantic models is equivalent to ordinary semantic validity. But if $\beta > \omega$, $\Phi = [\exists v_0 \phi_0 v_0 \wedge \ldots \wedge \exists v_n \phi_n v_n \wedge \ldots] \rightarrow [\exists v_0 \ldots v_n \ldots [\phi_0 v_0 \wedge \ldots \wedge \phi_n v_n \wedge \ldots]]$ is an example of a wff of $F_{\alpha\beta}$ which is semantically valid, but not valid in split semantic models. It follows that Φ is not provable in $\mathfrak{F}_{\alpha,\beta}$ nor in any system arising from $\mathfrak{F}_{\alpha,\beta}$ by addition of semantically valid α-propositional axiom schemata. An algebraic consequence is the existence of an α-complete Boolean algebra isomorphic to an α-field of sets having elements $b_{n\xi}$, $n < \omega$, $\xi < \alpha$, such that the indicated sums and products exist and $\prod_{n<\omega}\sum_{\xi<\alpha} b_{n\xi} \neq \sum_{f\in{}^{\omega}\alpha} \prod_{n<\omega} b_{nf(n)}$. (Notions of α-completeness and α-fields of sets are defined in terms of subsets of the algebra of power less than α.) (Received October 7, 1958.)

550-21. Solomon Feferman: Arithmetically definable models of formalized arithmetic.

Let L be a formalized first order language for arithmetic with primitive formulas x = y, σ(x,y,z) and π(x,y,z) (the latter two being given the usual interpretation x + y = z, x · y = z, resp.). A formula Ψ(x,y,z,...) of L is said to be a PR-formula if it is a translation, by Gödel's procedure, of a primitive recur-

sive relation $f(x,y,z,\ldots) = 0$. A PR-formula preceded by k quantifiers (of either type) is called a k-formula. A pseudo-model $\mathfrak{M} = \langle \omega, S, P \rangle$ is said to be k-definable if both S and P are definable by k-formulas; $\mathfrak{M}$ is said to be recursively enumerable if both S and P are definable by 1 (existential quantifier)-formulas. We write $\mathfrak{M} \equiv_k \mathfrak{N}$, where $\mathfrak{N}$ is the classical model, if $\mathfrak{M}$ satisfies all provable sentences of Peano's arithmetic together with all k-sentences true in $\mathfrak{N}$, and $\mathfrak{M} \equiv \mathfrak{N}$ if all sentences true in $\mathfrak{N}$ are true in $\mathfrak{M}$. Mostowski has raised the question (in Bull. Acad. Polon. Sci. Cl. III, vol. 5 (1957) pp. 705-710) whether there exist any recursively enumerable $\mathfrak{M} \equiv \mathfrak{N}$ for which $\mathfrak{M} \not\cong \mathfrak{N}$. The answer is given here as a special case of Theorem 1. If $\mathfrak{M}$ is arithmetically definable and $\mathfrak{M} \equiv \mathfrak{N}$ then $\mathfrak{M} \cong \mathfrak{N}$. This follows directly from: Lemma. If $\mathfrak{M}$ is k-definable and $\mathfrak{M} \equiv_{k+2} \mathfrak{N}$ then $\mathfrak{M} \cong \mathfrak{N}$. In the special case that $\mathfrak{M}$ is recursively enumerable we can improve this lemma to Theorem 2. If $\mathfrak{N}$ is recursively enumerable and $\mathfrak{M} \equiv_1 \mathfrak{N}$ then $\mathfrak{M} \cong \mathfrak{N}$. (Received October 7, 1958.)

550-22. R. C. James: Banach spaces with a specified number of separable conjugate spaces.

Let T be any Banach space with a basis $\{t_i\}$ for which $\sum_1^\infty a_i t_i$ converges if $\left|\sum_1^n a_i t_i\right|$ is a bounded function of n. It is shown that there is then a separable Banach space B such that B^{**} contains a subspace T_1 isometric with T and such that B^{**} is the linear span of T_1 and the natural image of B in B^{**}. This is used to show that, for each positive integer n, there is a separable Banach space B such that the nth conjugate space of B is the first nonseparable conjugate space of B. (Received October 8, 1958.)

550-23. Andrzej Ehrenfeucht: Theories having at least continuum many nonisomorphic models in each infinite power.

A sequence $\phi_1,\ldots,\phi_n,\ldots$ of formulas, with one free variable, x, is called independent in the (elementary) theory T if, for each finite sequence $\psi_1,\ldots,\psi_m$, where each ψ_i is either ϕ_i or $\sim\phi_i$, the formula $\forall x(\psi_1 \wedge \ldots \wedge \psi_m)$ is consistent in T. Theorem: If there is an independent sequence of formulas in the theory T, then for every power $\mathfrak{m} \geq \aleph_0$, there are at least $2^{\aleph_0}$ nonisomorphic models of T of power $\mathfrak{m}$. Let M be a model of a theory T. Write $x \sim y$ if and only if there is an automorphism f of M with $f(x) = y$. Denote the number of equivalence classes under $\sim$ by $\delta(M)$. Lemma: For any theory T (which has an infinite model) and any power $\mathfrak{m} \geq \aleph_0$, there exists a model M of power $\mathfrak{m}$ for which

$\delta(M) \leq \aleph_0$. Examples of theories satisfying the hypothesis of the theorem are: any extension of Peano's arithmetic, the theory of real closed fields, etc. (Received October 8, 1958.)

550-24. H. P. Flatt: Infinitely differentiable solutions of ordinary differential equations.

Let (*) denote the differential system $y' = f(x,y)$, $y(x_0) = y_0$, with $f(x,y)$ defined over the interval $I \equiv \{(x,y) \mid |x - y_0| \leq a, |y - y_0| \leq b\}$. By use of a dominant function constructed as the solution of a Stieltjes moment problem, it is shown that for certain classes of infinitely differentiable functions $f(x,y)$ of two real variables defined over I, the class to which the solution of (*) belongs may be determined, provided only that the solution of the Stieltjes moment problem exists. The proof depends on a refinement of an inverse function theorem for infinitely differentiable functions due to Liverman (Proc. N. A. S., v. 42 (1956) pp. 261-264). This result contains as a special case the classical results of Cauchy and Lindelöf for analytic functions of real variables. (Received October 8, 1958.)

550-25. Robert Osserman: Minimal surfaces with bounded normal direction.

Denote by $M(\alpha)$, for $0 < \alpha < \pi$, the class of simply-connected minimal surfaces having the property that the normal directions make an angle of at least α with some fixed direction. Note that if the surface S is of the form $z = f(x,y)$, then $S \in M(\pi/2)$. We have the following result: Let $S \in M(\alpha)$. Let p be a point of S, and let the geodesic distance from p to the boundary of S be at least d. Then the Gauss curvature K of S at p satisfies $|K| \leq f(\alpha)/d^2$, where f is an elementary function depending only on α and not on the particular surface. This generalizes a result of E. Heinz for surfaces of the form $z = f(x,y)$. If we take into account the direction of the normal at p we can improve this inequality in a manner analogous to E. Hopf's sharpening of Heinz' result. A corollary of our inequality is the following theorem: Every complete surface $S \in M(\alpha)$ is a plane. This was conjectured by L. Nirenberg and contains Bernstein's Theorem as a special case. (Received October 6, 1958.)

THE NOVEMBER MEETING IN DURHAM, NORTH CAROLINA

November 28-29, 1958

551-1. G. J. Minty: A general algorithmic method for routing and network problems.

The real variables $x_1,\ldots,x_n$ are to be chosen subject to bounds: $a_i \leqq x_i \leqq b_i$, and constraints: $\sum_{A_j} x_i = \sum_{B_j} x_i + c_j$, where A_j and B_j are two disjoint subsets of $(1,\ldots,n)$ and each i appears in at most one A_j and at most one B_j, so as to minimize $f_1(x_1) + \ldots + f_n(x_n)$, where the functions f_i are all concave-upward. The functions f_i are approximated by polygonal functions changing slope only at integer- (or m-decimal-place) values of their arguments; a hydraulic analogue is presented for the "polygonal" problem, which is then solved exactly in integers (if a solution exists) by an algorithm developed from the hydraulic analogue. The algorithm provides efficient means for introduction of perturbations (simultaneous changes in the conditions of the problem and in the solution) and for increasing the precision of the solution, without resolving from the beginning. Among the applications are: the Assignment and Transportation Problems, the Shortest-Route Problem, the Maximal Flow Through a Network Problem, electrical resistance-network problems, various nonlinear problems in Operations Research and fluid-flow networks, etc. (Received May 19, 1958.)

551-2. R. L. Plunkett: A topological proof of the continuity of the derivative of a function of a complex variable.

That the existence in a region of the derivative of a function of a complex variable implies the continuity of the derivative in that region is proved in this paper by elementary, or purely topological, means. That is, no use whatever is made of complex integrals or equivalent tools. (Received August 11, 1958.)

552-3. Robert Heller, Jr.: Some convergence theorems for continued fractions.

R. E. Lane and H. S. Wall have given [Trans. Amer. Math. Soc. vol. 67 (1949) pp. 368-380] a necessary and sufficient condition for convergence of continued fractions whose even and odd parts converge absolutely. This paper con-

tains a theorem which states a necessary and sufficient condition for absolute convergence of the even and odd parts of a continued fraction. This theorem is then applied to demonstrate the convergence of some particular continued fractions. If $0 < r < 1$, $0 < g_p < 1$, and $|a_p| - \mathrm{Re}\, a_p \leq 2r(1 - g_p)g_{p+1}$, $p = 1,2,3,\ldots$, then the continued fraction $1/1 + a_1/1 + a_2/1 + \ldots$ converges only if either (1) some $a_p = 0$, or (2) $a_p \neq 0$, $p = 1,2,3,\ldots$ and the series $\sum |c_p|$ diverges, where $c_1 = 1$ and $c_{p+1} = 1/(c_p a_p)$, $p = 1,2,3,\ldots$ [Wall, H. S., Analytic theory of continued fractions, D. Van Nostrand Company, Inc., New York, 1948, pp. 142-143]; in the reference just cited, Wall (p.143) offerd the conjecture that this might also be true even if $r = 1$. In this paper, an example is given which settles this conjecture in the negative. Some additional conditions which are necessary in order for the conjecture to be true are found and a new convergence theorem related to this conjecture is proved. (Received August 11, 1958.)

551-4. G. H. Meisters: On the almost periodicity of the integral of an almost periodic function.

Let $f(t)$ be a Bohr almost periodic function with the Fourier series $\sum A_n \exp(i\lambda_n t)$. Let $F(\alpha,t)$ be defined, for each positive real α, by the equation $F(\alpha,t) = \int_{-\infty}^{t} \exp(\alpha(s - t))f(s)ds$. Then for each positive α, $F(\alpha,t)$ is almost periodic in t with Fourier series $\sum A_n/(\alpha + i\lambda_n)\exp(i\lambda_n t)$, and satisfies the integral equation $F(\alpha,t) - F(\alpha,0) + \alpha\int_0^t F(\alpha,s)ds = \int_0^t f(s)ds$. From this equation it follows that if $F(\alpha,t)$ converges uniformly in t as α approaches zero and if $f(t)$ is almost periodic, then $\int_0^t f(s)ds$ is also almost periodic. By an application of the Bochner-Fejér summation method it can be shown that if $\sum |A_n|/\lambda_n^2$ converges, then $F(\alpha,t)$ converges uniformly in t as α approaches zero. Consequently we have the Theorem: If $\sum |A_n|/\lambda_n^2$ converges, then $\int_0^t f(s)ds$ is almost periodic. (Received September 15, 1958.)

551-5. D. O. Ellis: Some consequences of Wilson's theorem.

Certain consequences are deduced from the reknowned theorem of Wilson giving a necessary and sufficient test for primeness: (1) additional testors are exhibited which exemplify Wilson's theorem, (2) the testors are applied to obtain a novel result concerning the laplace probability that a randomly selected subset of a finite sequence is an initial interval of the sequence (the null set being counted an initial interval), (3) a formula is exhibited for $P(n)$, the number of primes not exceeding the positive integer $n \geq 2$. The methods are

strictly elementary. (Received September 22, 1958.)

551-6. C. C. Braunschweiger: Interior-like elements of cones. Preliminary report.

Let K be the positive cone of a partially ordered Banach space X. A point x in K, $x \neq \theta$, is an order-interior point if, for any y in K, $y \neq \theta$, there exists a z in K, $z \neq \theta$, such that both x and y are in $z + K$. A point x is a directionally-interior point of K if, for any line L in X through x, x is interior to $L \cap K$ relative to L. A quasi-interior point of K is a point x in K such that the linear extension of $K \cap (x - K)$ is dense in X. In this note we establish the following hierarchy classification of these interior-like points: Let K_i, K_0, K_d and K_q denote respectively the sets of all interior points, order-interior points, directionally-interior points and quasi-interior points of K. Then $K_d \subset K_q \subset$ and, if $K_i \neq \theta$ then $K_i = K_d = K_q$. Several examples are given to show that this is the strongest classification possible. (Received October 1, 1958.)

551-7. Tomlinson Fort: A problem in linear differential equations.

This paper is concerned with the problem relative to $u'' + (1 + \lambda g(x))u = 0$ suggested by R. E. Bellman in the Bull. Amer. Math. Soc. vol. 64 (1958) p. 68. Its chief result is that if $\mu_1(x)$ is that solution where $\mu_1(0) = 0$, $\mu_1'(0) = 1$, then $\mu_1(x)$ is asymptotic to $C_1(\lambda)\sin x + C_2(\lambda)\cos x$ where $C_1(\lambda)$ and $C_2(\lambda)$ are entire functions. If we write $C_1(\lambda)\sin x + C_2(\lambda)\cos x = r(\lambda)\sin(x + \theta(\lambda))$ then the zeros of $r(\lambda)$ are the only singularities of $\cos\theta(\lambda)$ and are all distinct from zero and are imaginary. (Received September 26, 1958.)

551-8. Sibe Mardešić: On a problem concerning local connectedness in the sense of singular homology.

In this paper a construction is described associating to every finitely presented group G a 2-dimensional continuous curve C(G) and a 1-dimensional continuous curve $D(G) \subset C(G)$ with the following properties. $\pi_1(C)$ is isomorphic to the (complete) direct product $\prod G$ of a sequence of copies of G. Every point of $C \setminus D$ has a neighborhood homeomorphic to the Euclidean plane and every point of D admits a basis of connected open neighborhoods U in C such that $U \subset C$ induces an isomorphism of $\pi_1(U)$ to a subgroup of $\pi_1(C)$ which is isomorphic to $\pi_1(C) \approx \prod G$. If G is a nontrivial finite perfect group, for example the alternating group A_n with $n > 4$, then one can derive from the above stated

properties that C(G) is lc_s^1 ($\mathcal{U}$-locally connected in the sense of singular homology) in all points but fails to be LC^1($\mathcal{U}$-locally connected in the sense of homotopy) in points of D(G). This solves the problem raised by H. B. Griffiths (Proc. London Math. Soc. 6 (1956) p. 479), to know whether the implication $LC^1 \Rightarrow lc_s^1$ can be reversed for metrizable compacta (of finite dimension). Conjecture: No metrizable compactum can be lc_s^1 at all points and fail to be LC^1 at precisely one point. (Received September 29, 1958.)

551-9. R. H. Bing and M. L. Curtis: Imbedding decompositions of 3-space in 4-space.

If a monotone decomposition of E^3 has two continua as its only nondegenerate elements, then it is easy to show that the decomposition space can be imbedded in E^4. The authors conjecture that if the nondegenerate elements are three circles such that each pair is linked, then the decomposition space cannot be imbedded in E^4. It is proved that if the nondegenerate elements are twelve planar circles, appropriately linked, then the decomposition space cannot be imbedded in E^4. The proof proceeds by showing that the decomposition space contains a certain 2-dimensional polyhedron which is not imbeddable in E^4. (Received October 3, 1958.)

551-10. M. K. Fort, Jr.: ϵ-mappings of a disc onto a torus.

Let X and Y be metric spaces, and let ϵ be a positive number. A mapping $f: X \longrightarrow Y$ is an ϵ-mapping if for each $y \in Y$, the diameter of $f^{-1}(y)$ is less than ϵ. In The Scottish book, S. Ulam asks the following question (problem number 21): If D is a closed disc and T is a torus, then does there exist for each $\epsilon > 0$ an ϵ-mapping of D onto T? It is shown in this paper that for sufficiently small ϵ, such mappings do not exist. (Received October 3, 1958.)

551-11. J. K. Hale: Criteria for orbital stability.

THEOREM. Consider the system of differential equations (1) $x_j'' + \sigma_j^2 x_j = \epsilon f_j(x,x',\epsilon)$, $j = 1,2,\ldots,n, x = (x_1,\ldots,x_n)$, where $\epsilon > 0$, each f_j is analytic function of x,x', ϵ in a neighborhood of $(0,0,0)$ in (x,x',ϵ) space, $\sigma_k > 0$, $m\sigma_1 - \sigma_k \neq 0$, $m\sigma_1 - 2\sigma_k \neq 0$, $\sigma_h \pm \sigma_k \neq m\sigma_1, k \neq h$, $k,h = 2,3,\ldots,n$, $m = 0,1,2,\ldots$. Let (2) $x_{10} = a\sigma_1^{-1}\sin(\omega t + \phi) + \epsilon W_1(\omega t + \phi,\epsilon)$, $x_{k0} = \epsilon W_k(\omega t + \phi,\epsilon)$, $k = 2,3,\ldots n$, be a periodic solution of (1), where $\omega = \omega(\sigma_1,\epsilon)$, $\omega(\sigma_1,0) = \sigma_1$, is analytic in ϵ at $\epsilon = 0$ and $W_j(\omega t + \phi,\epsilon)$ is analytic in ϵ at $\epsilon = 0$, periodic of period 2π in

$\omega t + \phi$, ϕ an arbitrary constant. If (3) $\int_0^{2\pi/\sigma_1} f_{kx_k'}(x_0, x_0', 0)\,dt < 0$, $k = 1,2,\ldots,$ where $x_0 = (x_{10},\ldots,x_{m0})$, then there exists an $\epsilon_0 > 0$ such that the solution (2) is orbitally stable for $\epsilon < \epsilon_0$. By evaluating a fourth order determinant for the characteristic exponents of the linear variational equation associated with (2), A. Andronov and A. Witt, [J. Exp. Theoret. Phys., USSR vol. 4 (1933) pp. 249-271] proved this theorem for $n = 2$. In this paper the theorem is proved for arbitrary n without using determinants, but by using a variant of the method of Cesari [Atti Accad. Italia Mem. Cl. Sci. Fis. Mat. Nat. vol. 11 (1940) pp. 633-692] to evaluate the characteristics exponents. The condition of analyticity of the differential system and the solution can be relaxed. (Received October 3, 1958.)

551-12. O. G. Harrold, Jr.: Euclidean domains with uniformly abelian local fundamental groups, III.

By the use of results of paper I of this sequence (Trans. Amer. Math. Soc. vol. 67 (1949) pp. 120-129) and the recently proved Dehn's lemma (C. D. Papakyriakopoulus, Annals of Math. vol. 66 (1957) pp. 1-26), it is shown that a 1-manifold in 3-space whose complement has the property implied in the title is tamely imbedded. By the use of a theorem recently established by R. H. Bing (Notices Amer. Math. Soc., vol. 5, No. 2, page 180) a similar theorem holds for 2-manifolds. Combining the two results gives the theorem: A manifold in 3-space is tamely imbedded if and only if each complementary domain has uniformly abelian local fundamental groups. This answers a problem proposed at the Summer Institute On Set Theoretic Topology, Madison, 1955 (Summary of Lectures and Seminars, page 56, problem 2). (Received October 3, 1958.)

551-13. S. M. Robinson: On the existence of the fundamental solution of the parabolic equation.

Consider the parabolic equation $L(u) \equiv \sum_{i,j=1}^{n} a_{ij}\partial^2 u/\partial x_i \partial x_j + \sum_{i=1}^{n} b_i \partial u/\partial x_i + cu - \partial u/\partial t = 0$, where the coefficients a_{ij}, b_i, and c are bounded and continuous functions of the variables $x_1,\ldots,x_n$; t and satisfy Hölder conditions with respect to the space variables, $x_1,\ldots,x_n$. Assume further that there exists two positive constants, g and G, such that $g\sum\lambda_i^2 \leq \sum a_{ij}\lambda_i\lambda_j \leq G\sum\lambda_i^2$. Let Ω be any measurable subspace of Euclidean n-space and T a positive constant. If the above conditions hold in $\Omega \times [0 \leq t \leq T]$, we show that there exists in this region a fundamental solution of the differential equation $L(u) = 0$. (Received October 3, 1958.)

551-14. Andrew Sobczyk: Stability of two-parameter systems.

A linear differential system has characteristic equation $c_n s^n + c_{n-1} s^{n-1} + \ldots + c_1 s + c_0 = 0$, (where s may be regarded as the variable of the Laplace transform). It is required to investigate stability of the system, in the case that the real coefficients c_j are functions of two real parameters x,y, that is, $c_j = h_j x + k_j y + b_j$, $j = 0,\ldots,n$. If $c_n = b_n > 0$, and $h_n = k_n = 0$, then the region of stability in the parameter plane is a sub-region of the region of positive coefficients; the latter region is a bounded or unbounded convex polygon. (In case $n = 2$, the two regions coincide.) Following a suggestion of R. Lanzkron, an algebraic curve is derived, in terms of squared frequency w^2 as parameter, a part of which forms a portion of the boundary of stability; the other portions are parts of the boundary of the convex polygon and parts of curves on which the Hurwitz determinants vanish. Examples are given for which the system is stable for no point in the parameter plane, although a region of positive coefficients exists; all possible cases are classified. (Received October 6, 1958.)

551-15. Jack Segal: Homogeneity of inverse limit spaces.

This paper is concerned with the question of homogeneity of inverse limit spaces. First it is shown that if the coordinate spaces are closed unit intervals and the bonding mappings are open, that the inverse limit space is not homogeneous. Second it is shown that if the coordinate spaces are arcwise connected, locally arcwise connected and homogeneous, and the bonding mappings are coverings, that the inverse limit space is homogeneous. (Received October 6, 1958.)

551-16. R. J. Koch: Arcs in partially ordered spaces.

In what follows, an arc will denote a continuum irreducibly connected between two points. The spaces involved are assumed to be Hausdorff, but not necessarily metrizable. The following theorem on the existence of arcs in partially ordered spaces is established: Let $(X, \leqq)$ be a compact partially ordered space and let W be an open set in X. If (1) $\{(x,y): x \leqq y\}$ is closed in $X \times X$ and (2) For any $x \in W$, each open set about x contains an element y with $y < x$, then any element x of W belongs to a compact connected chain C with $C \cap Fr(W) \neq \square$ and $x = \sup C$. Applications: (1) If S is a compact connected topological semigroup with unit u, then any neighborhood of u contains an arc. (2) A locally compact connected topological semigroup with zero, each of whose elements is idempotent, is arcwise connected. (3) If $(X, \leqq)$ is a compact

partially ordered space with closed graph and unique minimal element, and if $\{x: x \leqq a\}$ is connected for each $a \in X$, then X is arcwise connected. (Received October 6, 1958.)

551-17. A. T. James: Zonal functions of the representation of the linear group in the space of polynomials of a symmetric matrix. Preliminary report.

When a real positive definite matrix A is transformed by the real linear group, $G, A \to LAL'$, $L \in G$, the homogeneous polynomials of degree p in the elements of A constitute a representation of G whose character is the plethysm $\{2\} \circledast \{p\}$ of $\{2\}$ and $\{p\}$, which decomposes into single irreducible representations characterized respectively by each partition of 2p into even integers. The subspace in which an irreducible representation acts contains a unique polynomial which is invariant under the orthogonal group, called an elementary zonal function in the sense of E. Cartan, Oeuvres complètes, partie I, vol. 2, pp. 1045-1. The paper gives a method of calculating the elementary zonal functions. The representation $\{2\} \otimes \{p\}$ of G corresponds to the transitive permutation representation, D, of the symmetric group, S_{2p} on the set of distributions belonging to the partition 2^p of 2p; and, in particular, the zonal functions of G correspond to indecomposable idempotents of D, calculation of which thus yields the zonal functions. It is conjectured that the integral $\int (\mathrm{tr}(AHBH))^p dH$ taken over the orthogonal group, which arises in certain multivariate statistical distributions, is the sum of products of the elementary zonal polynomials in A and B. (Received October 6, 1958.)

551-18. M. K. Fort, Jr. and W. G. Horstman: Topological completeness of a space of homeomorphisms.

Let (X,d) be a compact metric space and let (Y,D) be a complete metric space. It is shown that if H is the set of homeomorphisms mapping (X, into (Y,D), then H with the metric defined by $\rho(f,g) = 1.\text{u.b.}\{D(f(x),g(x)) \; x \in X\}$ is topologically complete. This is accomplished by defining $F_n(f) = \text{g.l.b.}\{D(f(u), f(v)) | u \in X, v \in X, d(u,v) \geq 1/n\}$ and proving that H with the metric ρ^* defined by $\rho^*(f,g) = \rho(f,g) + \sum_{n=1}^{\infty} 2^{-n} \max[1, |\log F_n(f) - \log F_n(g)|]$ is complete and homeomorphic to (H,ρ). (Received October 6, 1958.)

551-19. S. L. Warner: Compact principal domains.

Theorem: If A is a compact principal domain, then either A is a finite field or A is the valuation ring of a locally compact field whose topology is given by a discrete valuation of rank one. (Received October 7, 1958.)

551-20. P. M. Swingle: Continua with sets of indecomposability.

Let M be a compact continuum and T and T_{xy}, $\supset x \cup y$, be subcontinua of M. We say T is region-containing if $T \supset$ an interior point of M. If $M \supset Z$ and every region-containing T of M contains Z and Z is the maximal set with this property, then we say Z is the set of indecomposability of M: existence is shown in Connected sets of Wada, Notices Amer. Math. Soc. April, 1958, p. 175. A composant T_p w.r.t. $p \in M$ is $[q|q \in M$ and there exists a non-region-containing T_{pq} of M]. If M has set Z as above then M is the disjoint union of uncountably many composants, where each T_p is dense in Z and $M - T_p$ is dense in M; thus for $p \in M$ there exist possible y densely in M such that each T_{xy} must contain Z; M contains densely a connected subset W such that, for each nondegenerate connected subset V of W, $\bar{V} \supset Z$, i.e. W is widely connected w.r.t. Z; if $Z \supset$ an open subset D, then W - D is totally disconnected. If M is a compact continuum and the union of subcontinua W_i each with a set of indecomposability Z_i ($i = 1,2,\ldots,n$; $n \geq 2$) and each $M_i \cap M_j$ ($i \neq j$) consists of a countable number of components, then there exist possible p_i densely in M_i such that any subcontinuum of M containing $\bigcup p_i$ must contain $\bigcup Z_i$. (Received October 7, 1958.)

551-21. R. A. Beaumont and R. J. Wisner: Integral domains defined on torsion-free groups of rank two.

A ring R is said to be defined on an abelian group G if a mapping $G \times G \rightarrow G$ is given which possesses the ring properties of associativity and distributivity over the operation in G. This paper is concerned with the case where G is a torsion-free group of rank 2. It is shown that R is a ring defined on G without proper divisors of zero if and only if R is isomorphic to a subring of a quadratic field extension $R(\alpha)$ of the rationals R. Moreover, G admits such a ring only if G is a subdirect sum of a rank 1 group with itself, and this rank 1 group cannot be of nil type. (Received October 8, 1958.)

551-22. E. J. Pellicciaro: On the compatibility of systems of linear differential equations.

Consider the system of differential equations described by the matrix equations $Y' + P(x,\lambda)Y = 0$, $A(\lambda)Y(x_1) + B(\lambda)Y(x_2) = 0$, where $A(\lambda)$, $B(\lambda) \in C$ on $[\lambda_1,\lambda_2]$ and $P(x,\lambda) \in C$ on $[x_1,x_2] \times [\lambda_1,\lambda_2]$. Application of elementary point set methods to the matrix solution $Y(x,\lambda)$ of the above yields the main theorem: If, for each λ of $[\lambda_1,\lambda_2]$, $Y(x,\lambda) \not\equiv 0$ on $[x_1,x_2]$, then $Y(x,\lambda) \in C$ on $[x_1,x_2] \times [\lambda_1,\lambda_2]$ if and only if the compatibility is constant there. (Received October 8, 1958.)

551-23. Lamberto Cesari: On total stability.

The equivalent concepts of Massera's total stability and Malkin's stability under constantly acting disturbances are considered. For systems of the form $x_j'' + \alpha_j x_j' + \sigma_j^2 x_j = \epsilon g_j(x,x',\epsilon)$, $j = 1,\dots,n$, $x_h' + \beta_h x_h = \epsilon g_h(x,x',\epsilon)$, $h = n+1,\dots,N$, $x = (x_1,\dots,x_N)$, $x' = (x_1',\dots,x_n')$, $\sigma_1 = 0$, $\sigma_j \geqq 0$, $j = 2,\dots,n$, $\beta_h > 0$, $\alpha_j \geqq 0$, $n \leqq N$, g_j satisfying solely a Lipschitz condition, $j = 1,\dots,N$, $\epsilon > 0$ small, existence theorems for a periodic solution Γ are given by the use of a method of successive approximations (L. Cesari, W. R. Fuller). Corresponding conditions for asymptotic orbital stability of Γ can be given by the same method (J.K. Hale). Under the same hypotheses, Γ is totally stable. For a fixed ϵ, under a small perturbation of the functions g_j, Γ may be replaced by a thin tube R (invariant surface) which is asymptotically stable from the outside. (Received October 8, 1958.)

551-24. R. P. Hunter: Certain one dimensional semigroups.

Let S denote a compact, connected, Hausdorff topological semigroup with unit u and zero 0. It is shown that if S is hereditarily unicoherent the unique irreducible continuum from 0 to u is an arc and a (commutative) subsemigroup. It then follows, from a result of Koch (Duke Math. J. vol. 24 (1957) pp. 611-616), that if S is one dimensional it is arcwise connected. A two dimensional compact, connected semigroup with no arc containing its unit element is constructed. (Received October 6, 1958.)

551-25. Jack Levine: Note on the number of pairs of nonintersecting permutations.

If $X_n(p,q) = x_1x_2\ldots x_n$, $Y_n(p,q) = y_1y_2\ldots y_n$ are two permutations of p a's and q b's, $(p + q = n)$, the pair (X_n, Y_n) is called a pair of nonintersecting permutations of length n, if X_n and Y_n have the property that the number of a's in the sequence $x_1x_2\ldots x_k$ is unequal to the number of a's in the sequence $y_1y_2\ldots y_k$ for $k = 1,2,\ldots,n-1$, $(0 < p < n, \quad 0 < q < n)$. The number $N_n(p,q)$ of such nonintersecting pairs is determined as $(N_n(p,q)(n-1)/pq)\binom{n-2}{p-1}\binom{n-2}{q-1}$, this being based on the idea of lattice permutations as developed by MacMahon. The total number N(n) of such pairs of length n is also found as $N(n)n^{-1}\binom{2n-2}{n-1}$. A generalization to sets of k nonintersecting permutations is stated. (Received October 14, 1958.)

551-26. R. A. Struble: On singular solutions of certain nonlinear differential equations from the theory of interior ballistics. Preliminary report.

We consider the autonomous system: $dx/dt = y$, $dy/dt = P$, $dz/dt = p^{\alpha}$ where $P = [z(1 + \theta - \theta z) - ky^2]/x$, $0 \leqq \alpha < 1$, $-1 \leqq \theta \leqq 1$ and $0 < k$. (cf. Theory of the interior ballistics of guns by J. Corner, John Wiley and Sons, Inc.,1950). The solutions emanating from the half line, $x = x_0 > 0$, $y = z = 0$, of singular points are discussed. Attention is given to the limit $x_0 \longrightarrow 0$ wherein for $\theta = -1$ there is obtained the interesting solution $P \equiv$ constant $\neq 0$. (Received October 14, 1958.)

551-27. J. W. Cell: Closed-form solutions of $z'' + (p + u)z' + (q + v)z = 0$ in terms of solutions of $y'' + py' + qy = 0$ and of known functions.

A sufficient theorem is established for closed-form solutions of $z'' + (p + u)z' + (q + v)z = 0$ in terms of solutions of the related equation $y'' + py' + qy = 0$ and of known functions. Solutions of the particular equation $z'' - i2\pi(1 + a/x^{1/2})z' - (\pi^2/S)z = 0$, where a and S are parameters, are expressed in terms of solutions of the simpler equations $y'' - i2\pi y' - (\pi^2/S)y = 0$ and parabolic cylinder functions or confluent hypergeometric functions. Properties of these solutions are established. (Received October 14, 1958.)

THE NOVEMBER MEETING IN EVANSTON, ILLINOIS

November 28-29, 1958

552-1. J. B. Butler, Jr.: A note on perturbation theory of ordinary differential operators. Preliminary report.

Let L_0 be an ordinary formally self adjoint differential operator of even order $n = 2\nu$ on the interval $0 \leq t < \infty$ (satisfying the conditions of Kodaira, Amer. J. Math. vol. 72 (1950) pp. 502-543) and let $L_\epsilon = L_0 + \epsilon q$ where $q(t)$ is a continuous decreasing real valued function and ϵ is a real parameter. Results of J. Moser for $n = 2$ (Math. Ann. vol. 125 (1952) pp. 366-393) are generalized in part as follows: Suppose the differential problem $L_0 u = \lambda u$, $[\phi_{0j}, u](0) = 0$, $j = 1,\ldots,\nu$ is self adjoint and that the following conditions hold, for ℓ in a fixed real finite interval Δ, (i) $\int_0^\infty \Phi^2(t)|q(t)|dt < \infty$, (ii) $\lim_{\delta \to 0+} |M^{jk}(\ell + i\delta)| < K$, $j,k = 1,\ldots,n$, $\ell \in \Delta$, where $\Phi(t) = \sup|s_j(t,\ell)|$, $j = 1,\ldots,n$, $\ell \in \Delta$, and $s_j(t,\ell)$ is a system of fundamental solutions of $L_0 u = \lambda u$ and M^{jk} is the characteristic matrix (for definitions see the above paper of Kodaira). Theorem. If L_0 and q satisfy (i), (ii) then the differential problem $L_\epsilon u = \lambda u$, $[\phi_{0j}, u](0) = 0$, $j = 1,\ldots,\nu$ is self adjoint for sufficiently small ϵ. Given any u vanishing outside a compact set the spectral measure $E^\epsilon(\Delta)$ of L_ϵ satisfies $E^\epsilon(\Delta)u = E^0(\Delta)u + \epsilon E^1(\Delta)u + O(\epsilon^2)$ where $E^0(\Delta)$ is the spectral measure corresponding to L_0 and $E^1(\Delta)$ is an integral operator. (Received September 17, 1958.)

552-2. Philip Dwinger: Completeness of quotient algebras of Boolean algebras. III.

Let A be a Boolean algebra and let I be an ideal of A. Let X be a disjointed subset of A/I. X is called strongly disjointed if for every $x \in A/I$ there exists an element $x' \in A$, such that $x'/I = x$ and such that the set $X' = \{x', x \in A/I\}$ is disjointed. Every countable disjointed subset of A/I is strongly disjointed. The following more general result is proved. (1) If (i) A is β-complete (β a cardinal number) (ii) I is α-complete for every $\alpha < \beta$, then every disjointed subset of A/I of at most β elements is strongly disjointed. The following is known. (2) If (i) A is a Boolean algebra and β a cardinal number (ii) ΣX exists for every disjointed subset X of A, $\kappa(X) \leq \beta$, then A is β-complete. (1) and (2) imply: (3) If (i) A is β-complete (ii) I is α-complete

for every $\alpha < \beta$ (iii) ΣX exists for every strongly disjointed subset X of A/I for which $\kappa(X) \leqq \beta$, then A/I is β-complete. (Received September 24, 1958.)

552-3. N. L. Alling: On an extension of a classical theorem on groups.

Let G be a (multiplicative) totally ordered group. Let its identity element be denoted by 1. Let e be in G, $e > 1$, and let $G_e = \bigcup_{n \in N}(e^{-n}, e^n)$. The author shows that there exists a mapping f of G_e into R such that $f(ab) = f(a) + f(b)$ and such that $f(e) = 1$. In case G is Archimedian, $G_e = G$ and $f^{-1}(0) = 1$; hence f is an isomorphism of G into R and G is Abelian. Thus the classical theorem due to Hölder can be obtained from the Author's result. (Received September 27, 1958.)

552-4. S. M. Shah: On meromorphic functions of finite order.

Let F(z) be a meromorphic function of finite positive order ρ. If g, g_1, g_2 be different meromorphic functions (or constants, finite or infinite), such that T(r) for each function is $o(T(r,F))$, and $I(r,F) = (1/r)\int_0^r \log^+ M(t,F)dt$, then it is shown that (1) $\liminf_{r\to\infty} I(r,F)/\{N(r,1/F - g_1) + N(r,1/F - g_2)\} < \infty$ if ρ be noninteger; and (2) $\liminf_{r\to\infty} I(r,F)/N(r,1/F - g) \leqq 3c(k)k^{\rho}/\rho$, where $k > 1$, c(k) is a constant depending on k, for every g with two possible exceptions. Similar relations with T(r,F) instead of I(r,F) have also been proved. (Received September 29, 1958.)

552-5. J. G. Wendel: Order statistics of partial sums.

Let X_n be identically and independently distributed, with c.f. ϕ; let S_n be their partial sums; for $k > n$ or $k \leqq n$ put $R_{n,k} = 0$ or k'th largest of $S_1^+, S_2^+, \ldots, S_n^+$, where $a^+ = \max(a,0)$. For the generating function of the joint c.f.'s $\zeta_{n,k}(\rho,\sigma) = E(\exp i(\rho R_{n,k} + \sigma S_n))$ we find the identity $\sum_{n\geq 0}\sum_{k\geq 1} w^n z^{k-1}\zeta_{n,k} = [(1 - z)(1 - wz\phi(\sigma))]^{-1} \exp \sum_{n\geq 1} n^{-1} w^n (1 - z^n)\psi_n(\rho,\sigma)$, where ψ_n is the joint c.f. of S_n^+ and S_n. This generalizes results of Pollaczek (C.R.Acad. Sci. Paris vol. 234 (1952) pp. 2334-2336) and Spitzer (Trans. Amer. Math. Soc. vol. 82 (1956) pp. 323-339). The method is that of "Spitzer's formula: a short proof" (to appear in Proc. Amer. Math. Soc.). The proof rests on the Banach-algebraic identity (*) $[I-wPF-wz(I-P)F]^{-1}e = \exp-[P \log (e-wf) + (I-P) \log(e-wzf)]$, wherein $f \in \mathfrak{M}$, a commutative B-algebra with identity e, w and z are sufficiently small complex numbers, I is the identity map, F is multiplication by f, and P is a

projection leaving e fixed and such that $P\mathfrak{M}$ and $(I-P)\mathfrak{M}$ are subalgebras. The special case of (*) with $z = 0$ already appears in loc. cit. The research was supported by USASADEA. (Received September 29, 1958.)

552-6. F. L. Spitzer: An extension of the renewal theorem.

The renewal theorem for random variables with positive expected value is extended to random variables whose expected value is zero. A particularly simple result is obtained for independent symmetric random variables which have a common probability density $k(x) = k(-x)$. Let $S_1, S_2, S_3, \ldots$ be the sequence of partial sums of the given random variables. Then one defines the renewal sequence $Z_1, Z_2, Z_3, \ldots$ as the following subsequence of $S_1, S_2, S_3, \ldots$: Z_1 is the first S_k which is positive, and Z_{n+1} is the first S_k which is greater than Z_n. It is shown that, if $\sigma^2 = \int_{-\infty}^{+\infty} x^2 k(x)dx \leqq \infty$, then $\lim_{x\to\infty} \sum_{n=1}^{\infty} \Pr[x < Z \leqq x+h] = h \cdot 2^{1/2}/\sigma$. This result is equivalent to a Tauberian theorem. It is known that there exists one and only one function $F(x)$ defined for $x > 0$, which (1) is non-decreasing, (2) has the value $F(0^+) = 1$, (3) satisfies the Wiener-Hopf equation $F(x) = \int_0^{\infty} k(x-y)F(y)dy$. It is shown that $\lim_{x\to\infty} [F(x+h) - F(x)] = h \cdot 2^{1/2}/\sigma$. The two problems and their answers are equivalent because, for $x > 0$, $F(x) = 1 + \sum_{n=1}^{\infty} \Pr[Z_n \leqq x]$. (Received October 2, 1958.)

552-7. Joseph Lehner: The Fourier coefficients of automorphic forms on horocyclic groups. II.

The author employs the circle method of Hardy-Rademacher to derive convergent series representations of the Fourier coefficients of automorphic forms. His main result is contained in the following THEOREM: Let Γ be a group of linear fractional transformations of the complex plane whose domain of discontinuity is the upper half-plane; let Γ possess translations and be finitely generated. Let $F(z)$ be an automorphic form of dimension $r > 0$ on Γ having the expansions $(-1/\lambda_k(z - p_k))^{-r} t_k^{\alpha_k} f_k(t_k)$, where $t_k = \exp(-2\pi i/\lambda_k(z - p_k))$ and $f_k(t) = \sum_{m=-\mu_k}^{\infty} a_m^{(k)} t^m$, at the parabolic points $p_k (1 \leqq k \leqq s)$. Then for $m \geq 0$, $a_m^{(k)} = (2\pi/\lambda_k)\exp(\pi i r/2) \sum_{j=1}^{s} \sum_{\nu=1}^{\mu_j} a_{-\nu}^{(j)} \sum_{c_{jk}} c_{jk}^{-1} A(c_{jk}, \nu_j, m_k) L(c_{jk}, m_k, \nu_j, r)$. Here L is a combination of elementary and Bessel functions, A is an exponential sum, and c_{jk} runs over certain entries in the matrix representation of the group Γ. This result has been obtained by Petersson by other methods (Math. Ann. vol. 127 (1954) pp. 33-81). (Received October 2, 1958.)

552-8. J. G. Thompson: Frobenius groups and a related problem.

The main result states that if p is an odd prime, and if the finite group G has the property $\mathfrak{N}_p$, then G is p-normal. The property $\mathfrak{N}_p$ is defined as follows: A group G has property $\mathfrak{N}_p$ if and only if G possesses a p-Sylow subgroup P, and P possesses a group of automorphisms $\mathcal{A}$ which includes the inner automorphisms and which can be lifted to automorphisms of G such that if $1 \neq H$ is any $\mathcal{A}$-invariant subgroup of P, then $N_G(H) = P$. As corollaries of this result it is shown that if the finite group G possesses a maximal subgroup which is nilpotent of odd order, then G is solvable. Also proved is the result that if a finite group G possesses an automorphism of prime order which leaves only 1 fixed, then G is nilpotent. The main tools in the proof include the Sylow theorems, Grun's second theorem, and results of P. Hall and G. Higman on p-solvable groups. (Received October 3, 1958.)

552-9. L. J. Heider: Compactifications of dimension zero.

Let X be a completely regular T_1 space. Let C(X) be the set of all real-valued functions, defined and continuous on X. For each element f of C(X), let $N(f) = (x:f(x) < 0)$. The condition requiring that each such set N(f) be a countable union of open-and-closed subsets of X is both necessary and sufficient in order that the Stone-Čech compactification space βX of X be of dimension zero. This condition may be restated in purely set-theoretic terms. Let $\mathfrak{M}$ be any family of open subsets of X. Let $\mathfrak{M}$ be called a completely regular family of open subsets if, for each element M of $\mathfrak{M}$ there are sequences $\{M_n'\}, \{M_n''\}$ of elements of $\mathfrak{M}$ with $M_n' \subseteq Co(M_n'') \subseteq M$ and $\bigcup_{n=1}^{\infty} M_n' = M$. It is then stated that the space βX is of dimension zero if and only if each member of any completely regular family of open subsets of X is a countable union of open-and-closed subsets of X. (Received September 30, 1958.)

552-10. W. A. Beyer: The divergence set of Rademacher series.

Let $R_i(x)$ $(0 < x \leq 1)$ $(i = 1,2,\ldots)$ be the ith Rademacher function and c_i $(i = 1,2,\ldots)$ be real numbers. Let $A = [x|\sum c_i R_i(x)$ converges $(0 < x \leq 1)]$ and $B = [x|\sum c_i R_i(x)$ diverges $(0 < x \leq 1)]$. μ denotes Lebesgue linear measure. It is known that $\mu(A)$ if and only if $\sum c_i^2 < \infty$. This paper shows that if $\mu(A) = 1$, then B is empty or has Hausdorff dimension 1 according as $\sum|c_i|$ is finite or infinite. The result also holds if $R_i(x)$ is replaced by $\cos 2^i \pi x$. (Received October 6, 1958.)

552-11. Bernard Friedland and T. E. Stern: On periodicity of states in linear modular sequential circuits.

The state periodicity of unexcited, linear modular sequential circuits characterized by the vector-matrix equation $s(n+1) = As(n)$ over GF(p) is considered by application of Galois field theory. It is shown that if the minimal polynomial $m'(x)$ of A of degree k' is irreducible, the set of all matrix polynomials in A of degree k' form a Galois field isomorphic to $GF[p,m'(\lambda)]$, and consequently that the state period is equal to the period (order) of the subgroup generated by λ in $GF[p,m'(\lambda)]$. If the minimal polynomial is reducible, it is shown that the matrix period is the least common multiple of the periods generated by λ in $GF[p,m_i(\lambda)]$ where $m_i(x)$ are the irreducible factors of the characteristic polynomial of A, and that every state period is a divisor of the matrix period. (Received October 6, 1958.)

552-12. S. V. Parter: On the eigenvalues and eigenvectors of a class of matrices.

Consider a real, square $n \times n$ matrix $A = (a_{ij})$. Let A satisfy the following two conditions: (i) $a_{ij} \neq 0$ if and only if $a_{ji} \neq 0$, (ii) $a_{ij}a_{ji} \geq 0$. Associate a linear graph G with A in the following manner. Consider n "points", $p_1, p_2, \ldots, p_n$. The point p_i should be directly connected by an arc to the point p_j $(i \neq j)$ if and only if $a_{ij} \neq 0$. Assume that in addition to (i) and (ii) A satisfies: (iii) G is a "tree". Then the following two theorems hold. Theorem A: The matrix A can be diagonalized with real eigenvalues by similarity transformation. Theorem B: A necessary and sufficient condition that a real number λ_0 be a multiple eigenvalue of A is: there should exist a point p of order ≥ 3 such that λ_0 is an eigenvalue of at least 3 of the matrices associated with the "branches" of the "tree" G which are connected to p. (Received October 6, 1958.)

552-13. R. H. Bing: Each homogeneous plane continuum that contains an arc separates the plane.

It is shown that each tree-like homogeneous plane continuum T contains an "end point" that is too sharp to belong to the interior of any arc. This "end point" p has the property that there is a positive number ϵ such that for each $\delta > 0$ there is a δ-chain $(d_1, d_2, d_3, \ldots, d_n)$ such that $p \in d_1$, the distance from

d_1 to d_2 is more than ϵ, and T intersects the boundary of $d_1 + d_2 + \ldots + d_n$ only in d_n. (Received October 6, 1958.)

552-14. E. V. Schenkman: On the norm of a group.

The norm N(G) of a group G has been defined by Baer to be the set of elements n of the group such that for every subgroup H of G, $nH = Hn$. It is shown here that N(G) is in the second center of G, and that the commutator subgroup of G is in the centralizer of the norm. (Received October 6, 1958.)

552-15. J. V. Talacko: On stochastic linear inequalities.

1. Given a linear system: $[a_{nm}][x_m] \leqq [c_n]$, subject to constrains $x_i \geqq 0$ and with objective $f(\sigma) \equiv x_{n+m+1} = \sum_i \sigma_i x_i$ to be an optimum (maximum) solution, where a, c and σ are parameters. By addition of m arbitrary variables we get an augmented simplex matrix and solution by simplex algorithm in the form $[\eta_{n,\overline{n+m+1}}][x_{m+n+1}] = [\mu_{n+1}]$ where the η matrix is a simplex resultant.
2. Problem and the solution: Let c_n in our system be statistics and a's and σ's parameters. If the distributions of c's are known, their confidence intervals may be found and we get a class of solutions. In particular, if c's are arithmetic means, in addition to their solutions, we may get two other solutions for upper and lower ends of intervals. If c's possess the Poisson's distribution, the method has been found very practical. (Received October 8, 1958.)

552-16. M. A. Martino, Jr.: A resolution of the identity for certain unbounded linear operators.

Let A be a scaler spectral operator, bounded or unbounded, on a Banach space $\mathscr{B}$, as defined by Bade (Pacific J. Math. vol. 4 (1954) pp. 373-392) such that the spectrum of A is contained in a band $|\mathscr{I}m(z)| \leqq \sigma_0$. (As an example if $\mathscr{B}$ is a Hilbert space one may take A to be any self-adjoint operator on $\mathscr{B}$.) Let B be any bounded operator on $\mathscr{B}$, not necessarily spectral, and let its sup norm be $\|B\|$. Then, denoting $R(\lambda, T) = (\lambda I - T)^{-1}$ for λ in the resolvent set of operator T, the operator-valued integral $(1/2\pi i)\int_{-\infty}^{\infty} [R(\rho - i\sigma, A + B) - R(\rho + i\sigma, A + B)]d\rho$ converges in the strong topology to the identity operator on $\mathscr{B}$ provided that $\sigma > \sigma_0 + \|B\|$. This result is used as a foundation for an operational calculus of functions of A + B. Applications are made to the solution of certain initial value problems for systems of parabolic differential equations. (Received October 8, 1958.)

552-17. L. F. McAuley: Local cyclic connectedness. I. Cyclic subelements of cyclic elements.

We shall say that a compact locally connected metric continuum M is locally cyclicly connected at p if and only if for each open set $D \supset p$, there exists a cyclicly connected open subset Q of M (each pair of points in Q lies on a simple closed curve in Q) such that $D \supset Q \supset p$. A subset of M is said to be an open cyclic subelement of M if and only if D is maximal with respect to the properties of being open and locally cyclicly connected at each point. The cyclic continuum $\overline{D}$ is called a true cyclic subelement. Other types of true cyclic subelements are studied along with sets analogous to the A-sets and H- sets due to G. T. Whyburn. A cyclic subelement theory somewhat similar to Whyburn's cyclic element theory is developed not only for Peano continua but also for aposyndetic continua. Relations between certain types of cyclic subelements and fine cyclic elements (due to L. Cesari and studied also by Neugebauer in Peano continua with finite degree of multicoherence) are pointed out. (Received October 8, 1958.)

552-18. G. Y. Rainich: Inversive geometry, complex numbers and proportionality.

A necessary and sufficient condition for the product of three inversions in circles to be an inversion is that the three circles belong to the same pencil. A necessary and sufficient condition for two circles to be orthogonal is that the inversions in these circles commute. In a euclidean plane π three points $0, U, \Omega$ are marked. Three types of double inversions (depending on relative positions of the two circles) are introduced as translations, rotations and stretchings. In terms of these and with the aid of the above propositions the points of $\pi - \Omega$ are converted into a field. The product of three inversions is considered as a special case of the proportionality operation $AB^{-1}C$ first introduced by Prüfer. Other special cases are considered. (Received October 8, 1958.)